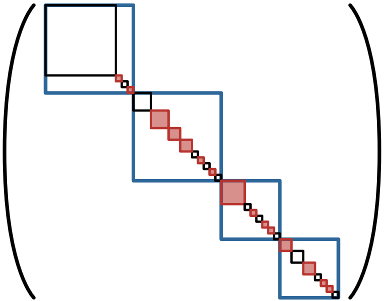
Institute for Theoretical Physics
High Performance Computational Physics

Anomalous Thermalization from Generalized Symmetries



Thea Budde

Syracuse Fermion Workshop 2026

arXiv:2604.12907

Motivation

Lattice model symmetries $\overset{?}{\longleftrightarrow}$ Continuum model symmetries

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What is a lattice symmetry actually? A different perspective:

- Statistical physics: The symmetries of the model are sufficient to predict the long time thermal behavior
- Ergodicity breaking: There are models in which it has been observed that "conventional symmetries" are not enough

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$$\text{Lattice model symmetries} \overset{?}{\longleftrightarrow} \text{Continuum model symmetries}$$

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- Statistical physics: The symmetries of the model are sufficient to predict the long time thermal behavior
- Ergodicity breaking: There are models in which it has been observed that "conventional symmetries" are not enough

Can we identify unconventional* lattice symmetries in these models?

*generalized and beyond

Outline

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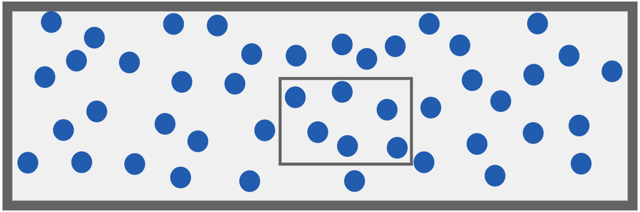
Will an isolated quantum system thermalize?



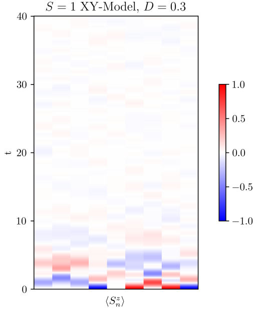
The large system acts as a heat bath for a small subsystem $\Rightarrow$ *Local* observables thermalize

Thermalization in isolated quantum systems

Non-integrable many-body Hamiltonians are expected to be ergodic and thermalize:

- After some time, in the thermodynamic limit, local observables have only small fluctuations
- Expectation values do not depend on details of initial state
- They can be predicted by an appropriate statistical ensemble

$$\langle \mathcal{O} \rangle_{\substack{t\to\infty \\ V\to\infty}} = \frac{1}{Z} \mathrm{Tr}\left(\mathcal{O}\rho_{\mathrm{thermal}}\right)$$



Deutsch, PRA 43, 2046 (1991); Srednicki, PRE 50, 888 (1994)

Ensembles from Ergodicity

The dynamics explore possible states uniformly subject to conserved quantities.

$\Rightarrow$ Maximum entropy distribution:

$$\rho_{\text{Gibbs}} = \frac{1}{Z} e^{-\beta H} e^{-\sum_i \mu_i G_i}, \qquad \langle\psi_0| G_i |\psi_0\rangle \equiv \mathrm{Tr}\left(G_i \rho_{\text{Gibbs}}\right)$$

But we should only consider G_i that affect local observables, not all $|E\rangle \langle E|$

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But we should only consider G_i that affect local observables, not all $|E\rangle \langle E|$

Which ones are these?

Conventional symmetries

U is relevant if it is a local observable itself

$$U = \prod_n u_n,$$

for example magnetization $U = e^{i\alpha M} = \prod_n e^{i\alpha\sigma_n^z}$.

More precisely: 0-form invertible symmetry

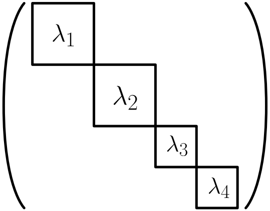
Symmetry sectors

Take a basis that diagonalizes a set of commuting conserved operators

$$U_i |\phi\rangle = \lambda_{i,\phi} |\phi\rangle .$$

Since $U_i(H|\phi\rangle) = \lambda_{i,\phi}(H|\phi\rangle)$, the Hamiltonian $H_{ab} = \langle\phi_a| H |\phi_b\rangle$ is block diagonal

$$H = \bigoplus_\lambda H_\lambda.$$



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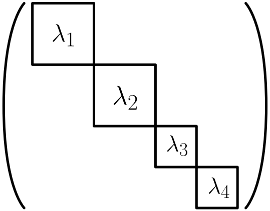
Therefore, sectors cannot mix under time evolution

$$e^{-iHt} = \bigoplus_\lambda e^{-iH_\lambda t}$$



Number of sectors with conventional symmetries

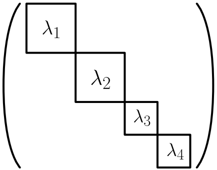
We take the Hilbert space to be a tensor product of identical finite-dimensional local Hilbert spaces

$$\mathcal{H} = h^{\otimes V} \qquad\qquad \dim(\mathcal{H}) = d^V$$

Then the number of conventional symmetry sectors grows at most polynomially with system size.

Proof: A local operator u_n has a finite number s distinct eigenvalues λ_i. Therefore $U = \prod_n u_n$ can have the $\binom{V+s-1}{s-1} = \mathcal{O}\left(V^{s-1}\right)$ distinct eigenvalues

$$\lambda = \lambda_1^{n_1} \lambda_2^{n_2} \ldots \lambda_s^{n_s}, \qquad n_1 + n_2 + \ldots + n_s = V.$$

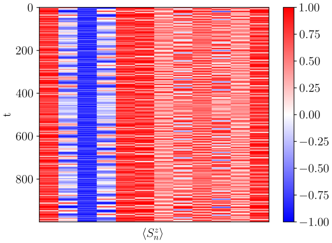


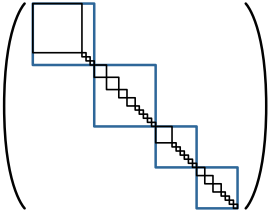
Hilbert Space Fragmentation

But for some models, the number of sectors in the product basis grows exponentially with volume:

$$H = \sum_n S_n^+ (S_{n+1}^-)^2 S_{n+2}^- + h.c.$$

Immediately ergodicity breaking and we observe localization:





S. Moudgalya et al., Rept.Prog.Phys. 85 (2022) 8, 086501

Outline

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Subsystem symmetries

There can be symmetries that are products over only a subsystem $\mathcal{M}$

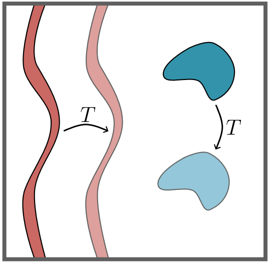
$$U(g) = \prod_{n \in \mathcal{M}} u_n(g)$$

Any disjoint translation is also an independent symmetry

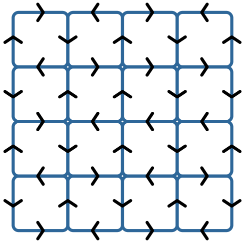
$$[H, T] = 0 \quad \Rightarrow \quad [TUT^{-1}, H] = 0$$

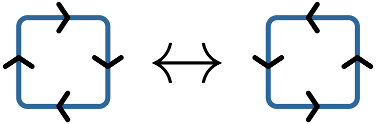
The sectors are now labeled by $|T|$ independent eigenvalues

$\Rightarrow$ Exponentially growing number of sectors



Example: U(1) gauge symmetry





$$H = \sum_{\square} S_1^+ S_2^+ S_3^- S_4^- + h.c.$$

$U(1)$ gauge theory conserves the Gauss law at each site:

$$G_n = \nabla E_n, \qquad U_n = e^{-i\alpha_n G_n}$$

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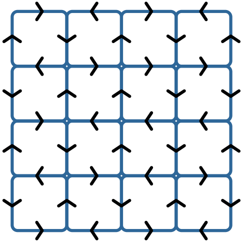

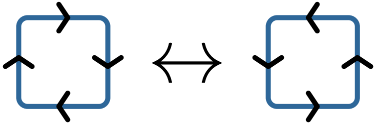

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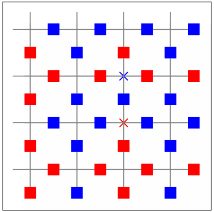
$\{G_n\}$ measures local static charge, which can have exponentially many configurations

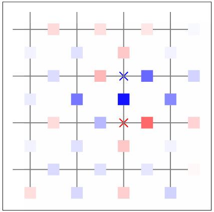
Constraint: Physical Hilbert space has $G_n \lvert\psi\rangle = 0$

Gauge symmetric Hamiltonians outside the physical sector

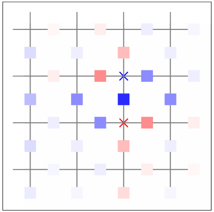
Non-physical sectors break translation invariance



$\langle S_n^z \rangle_{t=0}$



$\langle S_n^z \rangle_{t=10^4}$



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The constrained thermal ensemble ρ_{const} replicates this

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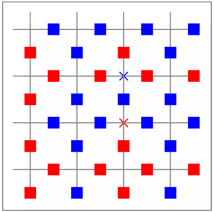
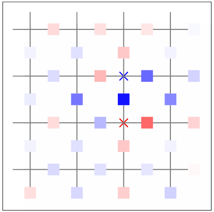


$\langle S^z_n \rangle_{t=0}$



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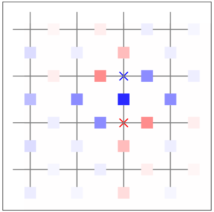


$\langle S^z_n \rangle_{\beta=0}$

The constrained thermal ensemble ρ_{const} replicates this

Disorder-free localization: Since most of the Hilbert space is non-physical and frozen, even the translation-invariant state $\frac{1}{\sqrt{2}^V}\left(|\uparrow\rangle + |\downarrow\rangle\right)^{\otimes V}$ results in anomalous correlations

A. Smith et al., Phys. Rev. Lett. 118, 266601 (2017)

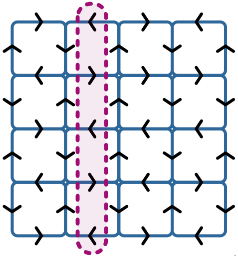
Higher-form symmetry

The magnetization of rows/columns is also conserved

$$W_x^{(1)}(y) = \sum_x S^z_{r,\hat{y}},$$

Within the physical sector, topological deformations are equivalent
$\Rightarrow$ 1-form symmetry with $\mathcal{O}(V)$ additional sectors

In the tensor-product basis, there are exponentially many sectors



Fragmentation from subsystem symmetries

Which known fragmented models can this mechanism explain?

- The PXP model

$$H = \sum_n P_n X_{n+1} P_{n+2} \qquad\qquad [H, |{\uparrow\uparrow}\rangle\langle{\uparrow\uparrow}|_{n,n+1}] = 0$$

- Most examples in higher dimensions
- Fragmentation in gauge theories [A. Ciavarella et al., Phys.Rev.D 112 (2025) 9, L091501]

<u>But there are some left:</u> t-J_z, $S = 1$ dipole conserving model...

Outline

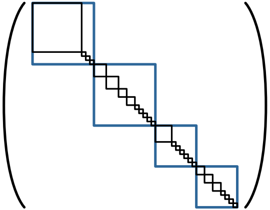
Partial isometries

Individual symmetry sectors may have additional symmetry

This can be described with a conserved operator

$$D = UP$$

- where $[D, H] = 0$
- U is unitary, but $[U, H] \neq 0$
- P is a projector onto a sector of H



Such conserved quantities are compatible with a generalized Wigner's theorem

G. Ortiz et al., Phys. Rev. B 113, 165126

Excursion: Non-invertible symmetry

The critical Ising model

$$H = \sum_n (X_n + Z_n Z_{n+1})$$

is obviously symmetric under $X_n \rightsquigarrow Z_{n-1}Z_n$, $Z_{n-1}Z_n \rightsquigarrow X_n$.

This cannot be implemented by a unitary operator

$$U\eta U^{-1} = U \prod_n X_n U^{-1} = \prod_n Z_{n-1} Z_n = 1 \quad \Rightarrow \quad \eta = 1 \lightning$$

only by a partial isometry $D = UP$, where P projects onto the parity-even sector.

N. Seiberg et al., SciPost Phys. 16 (2024) 064

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This model has the categorical symmetry

$$\eta^2 = 1, \quad D\eta = \eta D = D, \quad D^2 = (1+\eta)T$$

But not all partial isometries are necessarily non-invertible symmetries in the standard sense.

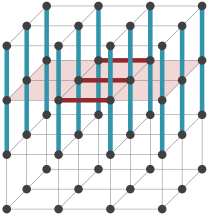
N. Seiberg et al., SciPost Phys. 16 (2024) 064

Higher form symmetry in one symmetry sector

Something similar happens in the gauge theory:

- The $(3+1)$d $U(1)$ pure gauge QLM has 1-form symmetries over planes

$$W_{xy}^{(1)}(z) = \sum_{x,y} S^z_{r,\hat{z}},$$



J. Steinegger et al., Phys.Rev.D 112 (2025) 11, 114512
TB, J. Pinto Barros, M. Marinkovic, arXiv:2604.12907

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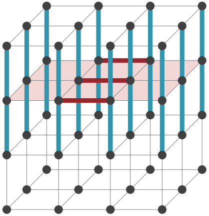
- Only in the $\pm\frac{L^2}{2}$ sectors, winding numbers are also conserved

$$W^{(2)}_{x}(y,z) = \sum_{x} S^z_{r,\hat{y}}$$

$\Rightarrow$ exponentially many sub-fragments even though $[H, W^{(2)}_x] \neq 0$



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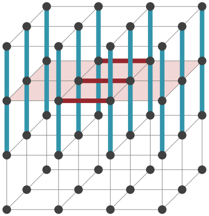
$\Rightarrow$ exponentially many sub-fragments even though $[H, W_x^{(2)}] \neq 0$

- $D = W_x^{(2)} P$ does commute with H



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Spin-1 dipole-conserving model

The Spin-1 dipole conserving model

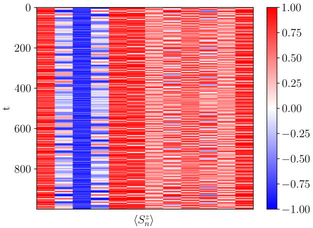
$$H = \sum_n S_n^+ (S_{n+1}^-)^2 S_{n+2}^- + h.c.$$

has many sectors that conserve

$$G_n = |0\rangle\langle 0|_n \,, \quad \tilde{G}_n = S_n^z + 2S_{n+1}^z + S_{n+2}^z$$

for even/odd sites.

This explains the localization, but not all fragments.



ID	G_{2n}	$\tilde{G}_{2n}$	G_{2n+2}	Compatible states $\lvert\sigma^z_{2n}\sigma^z_{2n+1}\sigma^z_{2n+2}\rangle$
1	0	-2	0	$\lvert -0-\rangle, \lvert +--\rangle, \lvert --+\rangle$
2	0	2	0	$\lvert +0+\rangle, \lvert ++-\rangle, \lvert -++\rangle$
3	0	-4	0	$\lvert ---\rangle$
4	0	4	0	$\lvert +++\rangle$
5	1	-3	0	$\lvert 0--\rangle$
6	0	-3	1	$\lvert --0\rangle$
7	1	3	0	$\lvert 0++\rangle$
8	0	3	1	$\lvert ++0\rangle$

TB, J. Pinto Barros, M. Marinkovic, arXiv:2604.15820

Open question: The t-J_z model

Spinful fermions with maximally repulsive on-site interactions:

$$\cdot \uparrow \cdot \cdot \cdot \downarrow \cdot \cdot \cdot \cdot \cdot \cdot \cdot \uparrow \cdot \cdot \cdot \cdot \cdot \uparrow \cdot \cdot \cdot$$

Flavor ordering is conserved, but there is no local observable to capture this.

Open question: The t-J_z model

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- Commutant algebra: Consider every quantity that commutes with each term in the Hamiltonian, regardless of locality [S. Moudgalya et al., Phys.Rev.X 12 (2022) 011050]

$$N^{\sigma_1 \sigma_2 \cdots \sigma_k} = \sum_{j_1 < j_2 < \cdots < j_k} N_{j_1}^{\sigma_1} N_{j_2}^{\sigma_2} \cdots N_{j_k}^{\sigma_k}.$$

Open question: The t-J_z model

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- Is there a more precise way of classifying useful conserved quantities?

  $\rightarrow$ Any function of locally measurable quantities, not only sums/products?
  $\rightarrow$ Categorical structure?

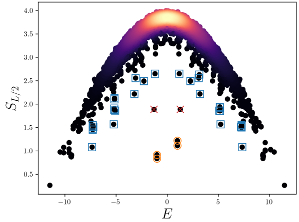
Outline

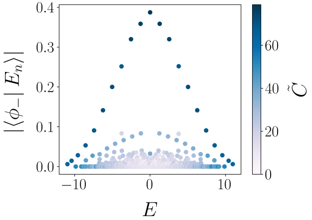
Quantum many-body scars (QMBS)

Typically, mid-spectrum states are thermal:

- Their expectation values reproduce the thermal distribution (ETH)
- Their entanglement is volume-law
- They have negligible overlap with product states

For QMBS this is not true:

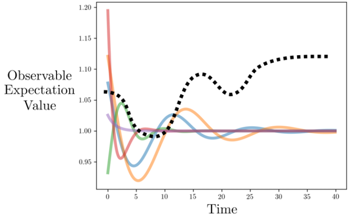


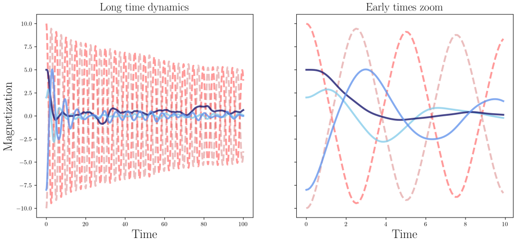


S. Moudgalya et al., Rept.Prog.Phys. 85 (2022) 8, 086501
J. Dong, **TB**, J. Pinto Barros, M. Marinkovic, in preparation

Scarred dynamics

The time evolution of states with large overlap with scars shows dynamical signatures:





TB, J. Dong, J. Pinto Barros, M. Marinkovic, Lattice 2025, arXiv:2604.05763
J. Dong, **TB**, J. Pinto Barros, M. Marinkovic, in preparation

Scars from Partial Isometries

Partial isometries were already used to construct generalized Hubbard models with exact revivals:

Many Body Scars as a Group Invariant Sector of Hilbert Space

K. Pakrouski[1], P.N. Pallegar[1], F.K. Popov[1], I.R. Klebanov[1,2]
[1]*Department of Physics, Princeton University, Princeton, NJ 08544 and*
[2]*Princeton Center for Theoretical Science, Princeton University, Princeton, NJ 08544*
(Dated: December 4, 2020)

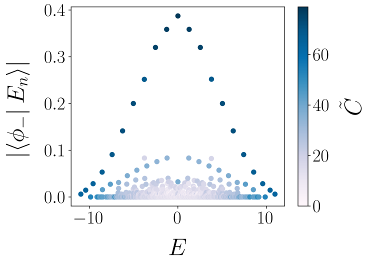
We present a class of Hamiltonians H for which a sector of the Hilbert space invariant under a Lie group G, which is not a symmetry of H, possesses the essential properties of many-body scar states. These include the absence of thermalization and the "revivals" of special initial states in time evolution. A particular class of

K. Pakrouski et al., PRL 125 (2020) 23, 230602

Scars from Approximate Symmetries

Weakly broken Lie algebra: Approximate $SU(2)$ algebra leads to revivals

- Scars have large overlap with the largest $\hat{C} = \vec{S}^2$ approximate symmetry sector



- Maybe an exact partial isometry exists too?

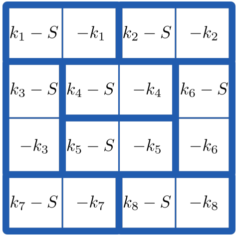
K. Bull et al., Phys. Rev. B 101, 165139 (2020)
J. Dong, **TB**, J. Pinto Barros, M. Marinkovic, in preparation

Exact quantum scars

- We found exact analytical expressions for some scars in $(2+1)d\ U(1)$ pure gauge theories:

$$|\psi_s^{(i,T)}\rangle = \frac{1}{(S+1)^{|T|/2}} \prod_{(n,n')\in T} \left(\sum_{k=0}^{S} (-1)^k (H_n^+)^{i-S+k} (H_{n'}^+)^{i-k} \right) |\mathbf{0}\rangle$$

- Many-body cage structure
- Valid for arbitrary volume and spin
- Efficient representation as MPS and future experimental preparation is feasible
- Related to the chiral and gauge symmetry?



TB, J. Pinto Barros, M. Marinkovic, Phys. Rev. D 110 (2024) 9, 094506, arXiv:2403.08892

Summary

- Ergodicity breaking can be a sign of an unconventional lattice symmetry
- We can explain several models' apparent ergodicity breaking with subsystem symmetries and partial isometries
- There are many more models that we don't know how to approach yet
- These models may be interesting from other viewpoints (definition of symmetry, phase diagrams, ...)

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Thank you!