Euclidean Correlation Functions in Quantum Gravity

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Einstein Hilbert Action

We want to see if we can make sense of the (Euclidean) path integral over the Einstein-Hilbert action using non-perturbative methods.

The hope is to make contact with Weinberg's asymptotic safety scenario, where the theory would be renormalizable nonperturbatively.

Continuum Euclidean path-integral:

$$Z = \int \mathscr{D}g \; e^{-S[g]}, \tag{1}$$

$$S[g_{\mu\nu}] = -\frac{2}{\kappa^2} \int d^d x \sqrt{\det g}(R - 2\Lambda), \tag{2}$$

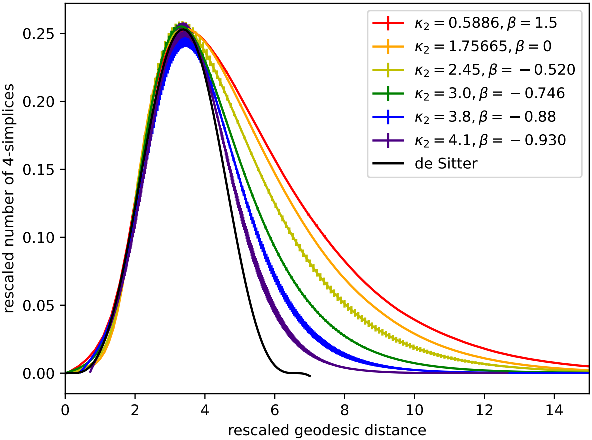
where $\kappa^2 = (32\pi G)$.

We focus on Euclidean Dynamical Triangulations, a discretized formulation of gravity.

Main problems to overcome

- Must show recovery of semiclassical physics in 4 dimensions, reproducing general relativity in the appropriate limit.
- Must show existence of continuum limit at continuous phase transition. This would realize asymptotic safety.
- Must make contact with experiment.

Emergence of de Sitter space



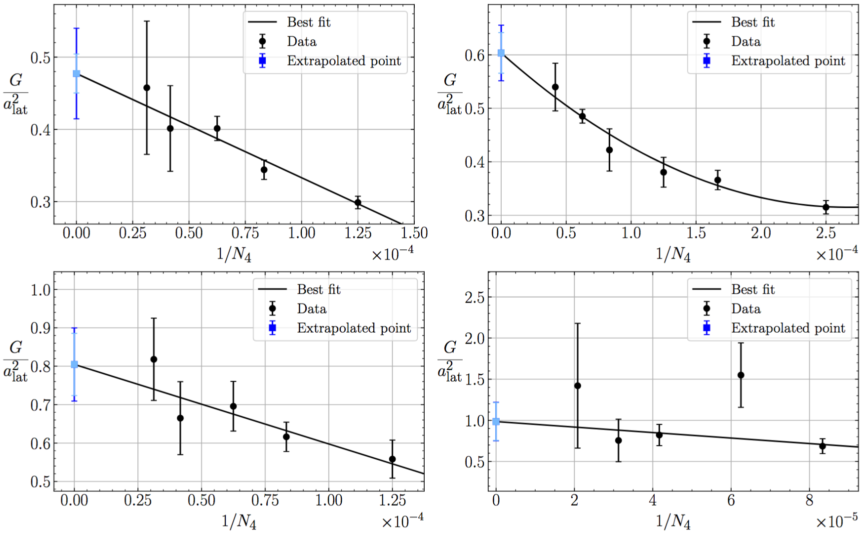
Physical phase can be simulated with a prescription for tuning the bare parameters. As the lattice spacing becomes finer, the lattice results approach the classical curve for Euclidean de Sitter.

Determining the lattice spacing

Volume expectation values can be used to extract Newton's constant. Assumes de Sitter solution dominates the partition function. Leads to expression for G, given expression for de Sitter entropy.

M. Dai, W. Freeman, JL, M. Schiffer, J. Unmuth-Yockey, PRD 111 (2025) 3, 034514, arXiv:2408.08963.

Values of Newton's constant



Euclidean Correlation Functions

Natural to look at 2-point functions of geometric observables, like curvature-curvature and volume-volume correlation functions.

$\langle R(y)R(x)\rangle$ and $\langle V(y)V(x)\rangle$ can be constructed straightforwardly in EDT.

They can also be calculated in the low energy effective theory of gravity. Although the EFT is non-renormalizable, low order calculations often give universal predictions, involving only Newton's constant.

Challenges of perturbative gravity

Former grad student Kenny Ratliff and I completed the EFT calculation (about flat space) through next-to-leading order.

JL and K Ratliff, Phys. Rev. D 113 (2026) 10, 106032 (arXiv:2510.11888)

Challenges involve:

- Feynman rules are complicated. Three-graviton vertex has close to two hundred terms.
- Coordinate corrections must be included. Source-sink separation most conveniently expressed in terms of distances on flat background, but quantum fluctuations of spacetime must be taken into account. Diffeomorphism invariant observables, and thus the coordinate corrections, are necessarily non-local.

Gravity Action

$$S = S_{\text{EH}} + S_{\text{gf}} + S_{\text{gh}} + S_{R^2}. \tag{3}$$

The Einstein-Hilbert action is

$$S_{\text{EH}} = -\frac{2}{\kappa^2} \int \mathrm{d}^d x \sqrt{-g} R, \tag{4}$$

Expanding about flat space, we have $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$, with $\eta_{\mu\nu}$ the Minkowski metric.

Gauge-fixing and ghost action

The gauge-fixing action is

$$S_{\rm gf} \quad = \tfrac{1}{2\alpha} \int \mathrm{d}^d x \left(\partial_\nu h^{\mu\nu} - \tfrac{1+\beta}{d} \partial^\mu h\right) \left(\partial^\lambda h_{\mu\lambda} - \tfrac{1+\beta}{d} \partial_\mu h\right), \tag{5}$$

with α and β gauge-fixing parameters. This is the generalized de Donder gauge. The Fadeev-Popov gauge-fixing procedure requires a Jacobian in the path integral that can exponentiated, where it becomes the ghost action. This ghost action is

$$\begin{aligned} S_{\rm gh} = \int \mathrm{d}^d x \Bigg\{ &\bar{c}^\mu \partial^2 c_\mu + \left(1 - \tfrac{2(1+\beta)}{d}\right) \bar{c}^\mu \partial_\mu \partial^\nu c_\nu \\ +\kappa \Bigg[&\left(1 - \tfrac{2(1+\beta)}{d}\right) \bar{c}^\mu h_{\nu\rho} \partial^\rho \partial_\mu c^\nu - \tfrac{1+\beta}{d} \left(\bar{c}^\mu \partial_\mu c^\nu \partial_\nu h + \bar{c}^\mu c^\nu \partial_\mu \partial_\nu h\right) \\ &- \tfrac{2(1+\beta)}{d} \bar{c}^\mu \partial_\mu h_{\nu\rho} \partial^\rho c^\nu + \bar{c}^\mu \partial_\mu c^\nu \partial_\rho h_\nu{}^\rho + \bar{c}^\mu c^\nu \partial_\nu \partial_\rho h_\nu{}^\rho \\ &+ \bar{c}^\mu h_{\mu\nu} \partial^2 c^\nu + \bar{c}^\mu \partial_\nu h_{\mu\rho} \partial^\rho c^\nu + \bar{c}^\mu \partial_\rho h_{\mu\nu} \partial^\rho c^\nu \Bigg] \Bigg\}, \end{aligned} \tag{6}$$

$\mathcal{O}(R^2)$ action

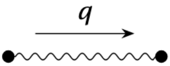
We need the higher derivative operators to the order we work.

$$S_{R^2} = \int \mathrm{d}^d x \sqrt{-g} \left(c_1 R^2 + c_2 C^2 + c_3 G \right), \tag{7}$$

where $C^2 = R^{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta} - 2R^{\mu\nu} R_{\mu\nu} + \frac{1}{3} R^2$ is the square of the Weyl tensor, and $G = R^{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta} - 4R^{\mu\nu} R_{\mu\nu} + R^2$ is the Gauss-Bonnet term.

The Gauss-Bonnet term is a topological invariant in 4 dimensions, so we could ignore it, except that it can lead to evanescent contributions in dimensional regularization. It turns out not to contribute to the things we care about here.

Curvature-curvature correlator at tree-level



$$= \mathrm{i}A_{\mathscr{R}} = -\frac{3}{2}\mathrm{i}\kappa^2 q^2,$$

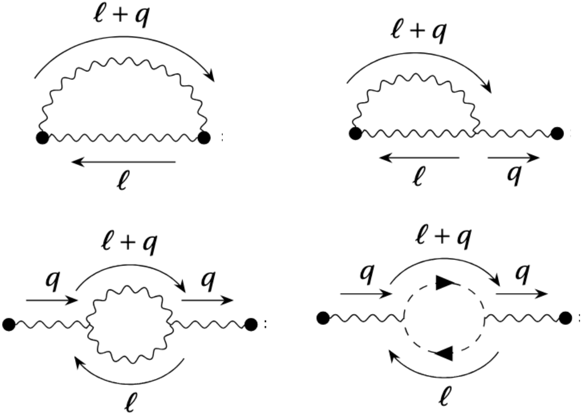
Fourier transforming to position space, we have

$$\int \frac{d^4q}{(2\pi)^4} iA_{\mathscr{R}} e^{-iq\cdot x} \to \frac{3\kappa^2}{2}\partial_E^2 \delta^4(x_E), \tag{8}$$

which vanishes away from the origin [G Modanese, Phys. Lett. B 288, 69 (1992)].

Vanishes, as far as lattice is concerned.

Curvature-Curvature correlator at one-loop



$$iA^{\text{one}-\text{loop}}_{\mathscr{R},\text{stand}} = \frac{\kappa^4 q^4 \beta}{4(\beta-3)^3}\left[(2\beta^2+5\beta-9)\,B_0(q^2,0,0) - 3(\beta-1)\,q^2\,C_0(0,q^2,q^2,0,0,0)\right],$$

with B_0 and C_0 Passarino-Veltman scalar integrals. Note that gauge dependence on β does not cancel!

Coordinate corrections

We transform into coordinates that are the “straightest” possible, where the curvature vanishes,
[R. Brunetti, K. Fredenhagen, T.-P. Hack, N. Pinamonti, and K. Rejzner, JHEP 08, 032 (2016), arXiv:1605.02573],
[M. B. Fröb, Class. Quant. Grav. 35, 055006 (2018), arXiv:1710.00839]

$$\nabla^2 X^\mu = \frac{1}{\sqrt{-g}} \partial_\alpha \left(\sqrt{-g} g^{\alpha\beta} \partial_\beta X^\mu \right) = 0. \tag{9}$$

The X can be expanded perturbatively,

$$X = \sum_{a=0}^{\infty} \kappa^a X_a. \tag{10}$$

The first non-trivial order leads to

$$\nabla^2 X_1^\mu = J_1^\mu, \tag{11}$$

with

$$J_1^\mu = \partial_\alpha h^{\alpha\mu} - \frac{1}{2} \partial^\mu h. \tag{12}$$

This has the solution

$$X_1^\mu(x) = \int d^d x' \, G(x, x') J_1^\mu(x'). \tag{13}$$

Coordinate corrections to curvature operator

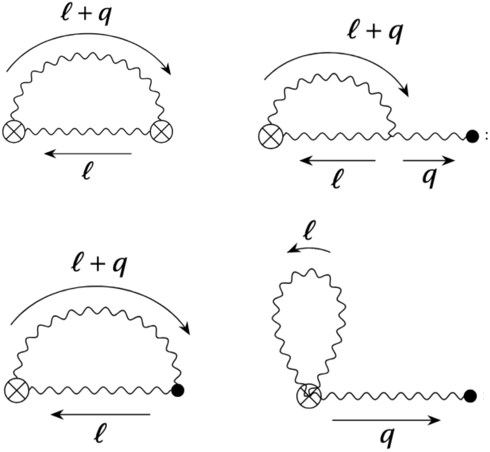
Can work out how these enter various quantities, including the scalar curvature,

$$R = \kappa R_1 + \kappa^2 \left(R_2 - X_1^\alpha \partial_\alpha R_1 \right) + \kappa^3 \left(R_3 - X_1^\alpha \partial_\alpha R_2 + \frac{1}{2} X_1^\alpha X_1^\beta \partial_\alpha \partial_\beta R_1 \right.$$
$$\left. + X_1^\alpha \partial_\alpha X_1^\beta \partial_\beta R_1 - X_2^\alpha \partial_\alpha R_1 \right) + O(\kappa^4), \tag{14}$$

with R_1, R_2, etc., the standard expressions for the expansion of the scalar curvature to a given power of h.

X_1 and X_2 are coordinate corrections. Note that they involve an integral over $G(x', x)$, which in momentum space takes the form of a scalar propagator. Thus the operator becomes nonlocal.

Coordinate correction diagrams at one-loop



$$iA^{\text{one-loop}}_{\mathscr{R},\text{cc}} = \frac{\kappa^4 q^4}{16(\beta-3)^3}\Bigg[\left(-9\beta^3-11\beta^2+9\beta+27\right)B_0\left(q^2,0,0\right)$$

$$+12\beta\left(\beta-1\right)q^2 C_0\left(0,q^2,q^2,0,0,0\right)\Bigg]$$

Result

Adding all 8 diagrams, we get

$$iA_{\mathscr{R}}^{\mathrm{one-loop}} = -\frac{\kappa^4}{16} q^4 B_0\left(q^2, 0, 0\right), \tag{15}$$

The β dependence cancels!

Fourier transforming, we get for the position space curvature correlator

$$\mathscr{F}\left(iA_{\mathscr{R}}\right) = \frac{768 G^2}{\pi^2 x_E^8} + O(G^3). \tag{16}$$

Following a similar calculation for the volume-volume correlator we get

$$\mathscr{F}\left(iA_{\mathscr{G}}\right) = \frac{12 G}{\pi x_E^2} + \frac{4 G^2}{\pi^2 x_E^4} + O(G^3). \tag{17}$$

Adding scalar fields and going to (partial) NNLO

$$\mathscr{F}(iA_{\mathscr{R}}) = \frac{768G^2}{\pi^2 x_E^8}\left(1+\frac{N_s}{2}\right) + \frac{3,538,944G^3}{\pi x_E^{10}}\left[\alpha_{\mathscr{R}}^{\mathrm{two-loop}} + N_s\left(c_1 + s_2 - \frac{1}{3}s_3\right)\right] + O(G^4)$$

$$\mathscr{F}(iA_{\mathscr{G}}) = \frac{12G}{\pi x_E^2} + \frac{4G^2}{\pi^2 x_E^4}\left(1+\frac{N_s}{2}\right) + \frac{3072G^3}{\pi x_E^6}\left[\alpha_{\mathscr{G}}^{\mathrm{two-loop}} + N_s\left(c_1 + s_2 - \frac{1}{3}s_3\right)\right] + O(G^4),$$

Can add N_s real scalar fields, minimally coupled to gravity. New LEC's show up at $\mathscr{O}(G^3)$.

The s_i are higher derivative couplings of scalars to gravity.

The two-loop terms $\alpha^{\mathrm{two-loop}}$ are unknown, but should be order one numbers.

The LEC c_1 is the coefficient of the R^2 operator.

(JL and K Ratliff, in preparation)

EDT simulations

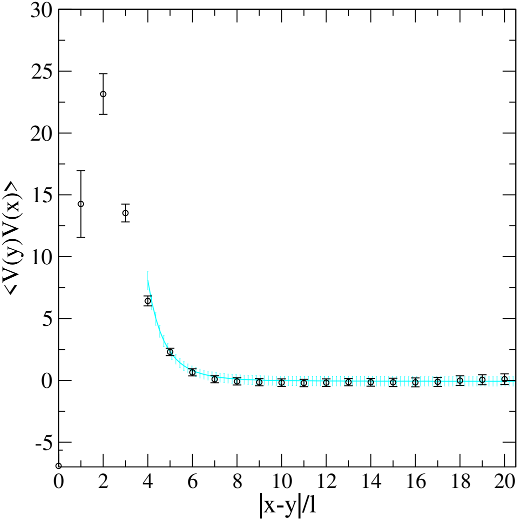
New numerical simulations performed with EDT minimally coupled to one real scalar field.

Made use of Walter Freeman's rejection free algorithm for the geometry with heat bath updates for the scalar field.

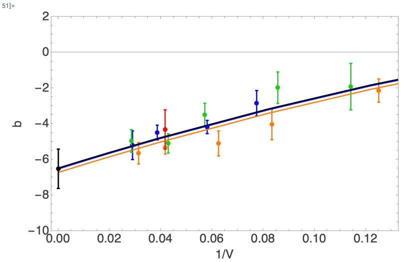
Calculated relative and absolute lattice spacings as in previous work, and found emergent de Sitter space similar to pure gravity.

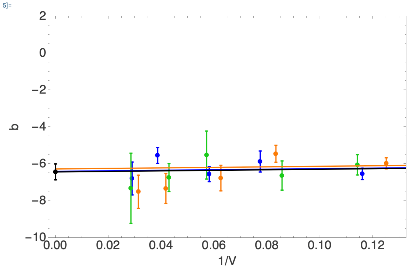
Proceeded to calculate $\langle V(y)V(x)\rangle$ and $\langle R(y)R(x)\rangle$ correlation functions.

Volume-Volume correlation function



Power law



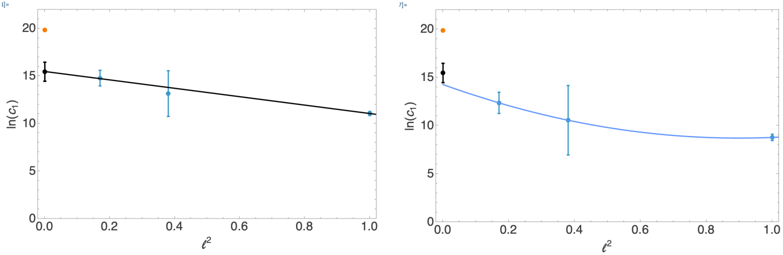


Volume dependence of exponent in power law fit (to the form $Ax^b + c$) at several lattice spacings.

Two different discretizations of the local volume operator. Left is the simplex and right is the triangle.

Power law is not compatible with EFT prediction of -2. If we assume a large subleading LEC, we can extract its value from the normalization.

Coefficient of R^2 term



Can extract the coefficient of the R^2 operator from the normalization of the correlator. Assume the power law and normalization hold out to large distances until this formally subleading (in G/x^2) term actually becomes subleading. Then its normalization leads to the value of c_1 in the EFT. (Weak scalar binding rules out large s_i.)

Two different discretizations of the local volume operator. Left is the simplex and right is the triangle.

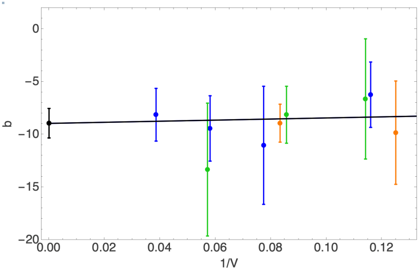
R^2 Inflation

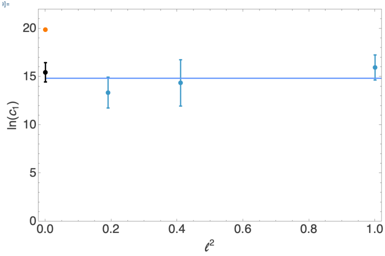
A field transformation of the metric transforms the R^2 term to a scalar field with potential

$$V(\phi) = V_0 \left(1 - e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_{\mathrm{Pl}}}} \right)^2 \tag{20}$$

with V_0 inversely proportional to c_1. This model is a good description of our universe, with V_0 tuned to match the amplitude of density fluctuations in the cosmic microwave background (CMB).

Curvature-curvature correlator





Can also extract the coefficient of the R^2 operator from the normalization of the curvature-curvature correlator. In the R^2 theory the power law fall off is 10. Compatible with data at low precision.

Assume it is 10 when computing c_1 from normalization. Gives agreement with volume-volume correlator.

Outlook

Two finer lattice spacings with a massless scalar in progress. Will help solidify errors on continuum extrapolation, especially on coefficient of R^2 term.

Most of the Standard Model fields are quenched. Can add a more realistic matter sector, first ignoring masses and matter interactions (SM is weakly coupled at the Planck scale). This is in progress.

Appears that R^2 inflation is a consequence of EDT, with no adjustable parameters put in by hand. Inflationary potential, including normalization, is a prediction. In this EDT world, with one scalar field, amplitude of density fluctuations in CMB are $\mathscr{O}(10^{-4})$. In real world they are $\mathscr{O}(10^{-5})$.

Must unquench!