What Does a QCD Wilson Line Flow to in the IR?

Ho Tat Lam

USC University of Southern California

Based on [HTL 260X.XXXX]

New Developments in Lattice Fermions @ Syracuse 2026

QCD & Chiral Lagrangian

[..., Witten '83, ...]

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Consider 3+1d $SU(N)$ QCD + N_f massless Dirac fermions in the fundamental rep

$$S = \int d^4x \left[\frac{1}{4g^2} \mathrm{Tr}(F^{\mu\nu}F_{\mu\nu}) + i \sum_{i=1}^{N} \bar{q}_i \gamma^\mu D_\mu q_i \right]$$

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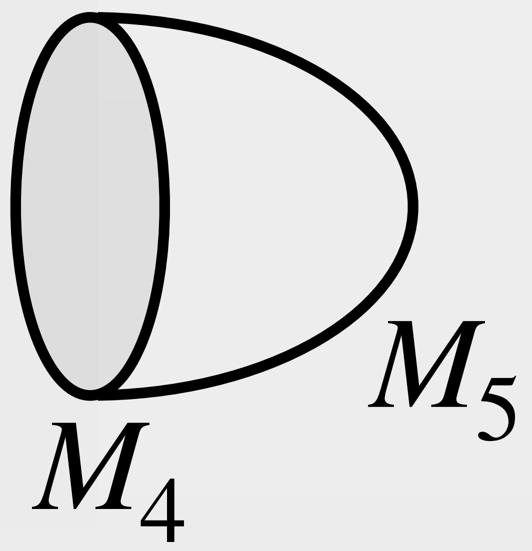

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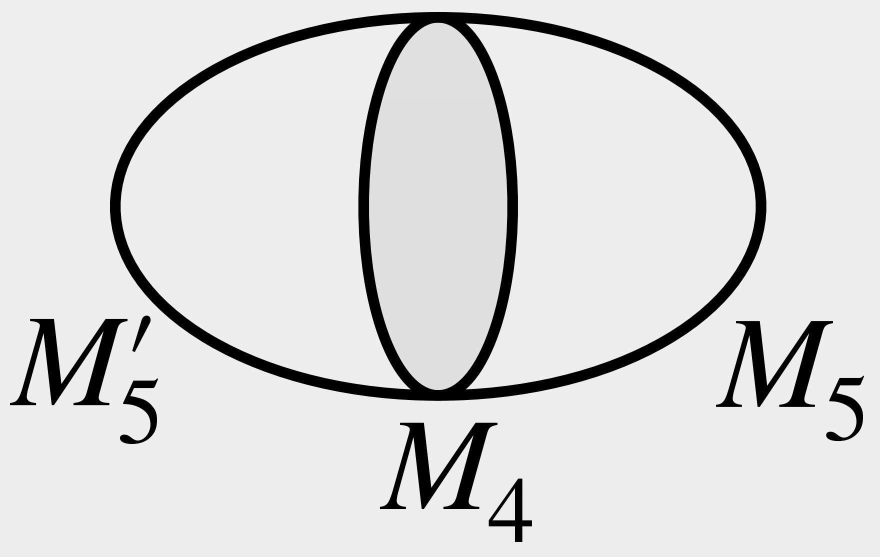

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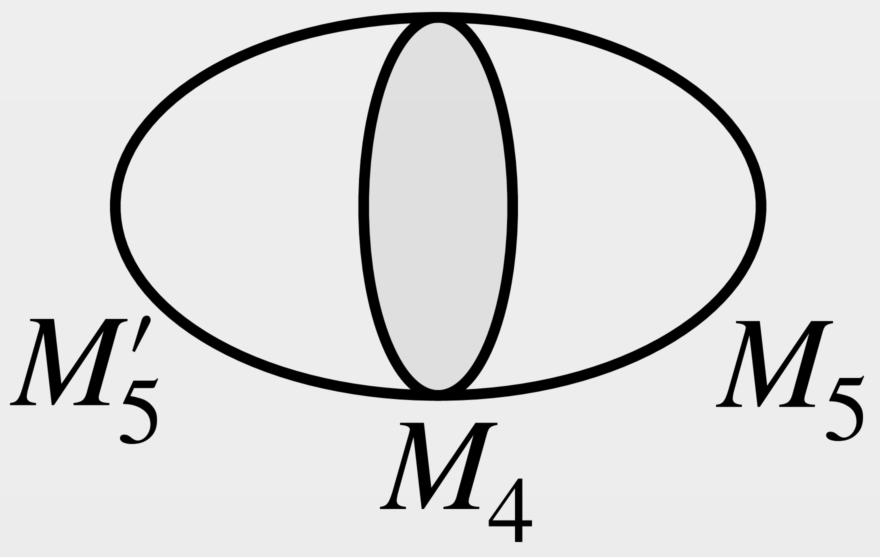

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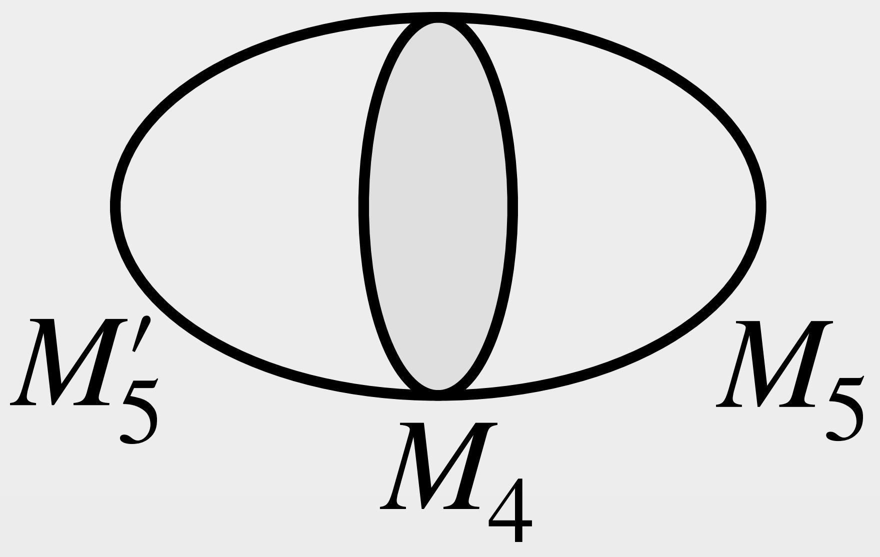

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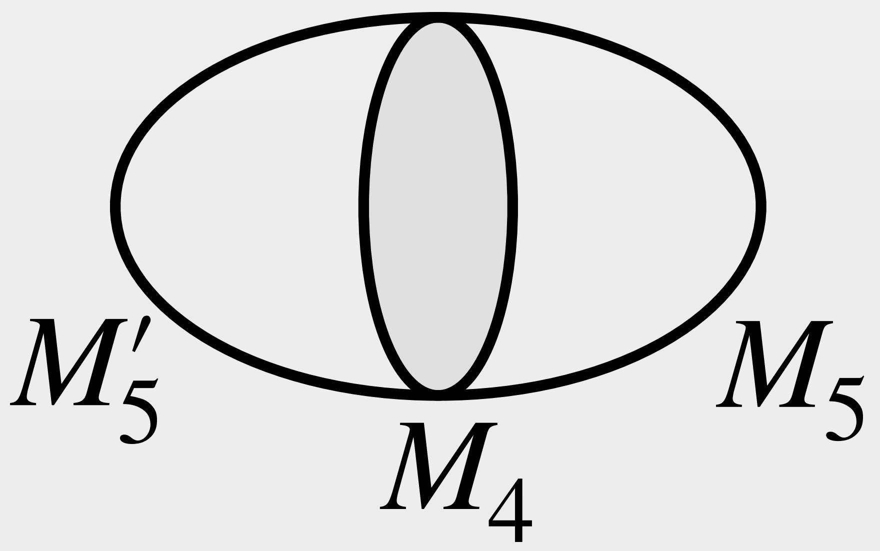

$$J_B = \frac{1}{24\pi^2} * \mathrm{Tr}[(U^\dagger dU)^3]$$

Baryon $\epsilon_{a_1 \cdots a_N} \bar{q}_{i_1}^{a_1} \cdots q_{i_N}^{a_N}$

$\rightarrow$ Skyrmion

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Why? Maybe because one may intuitively expect a boring answer …

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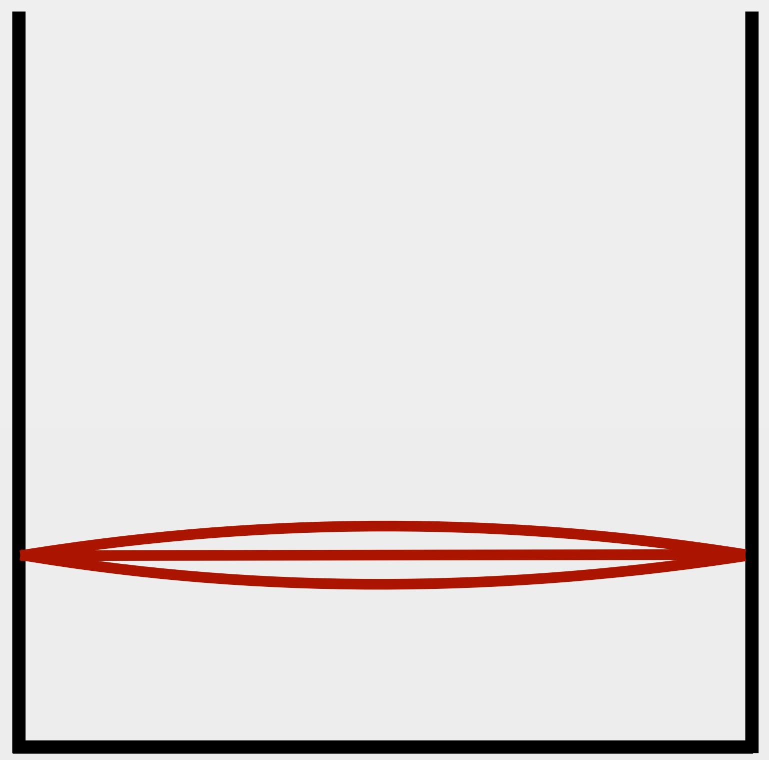

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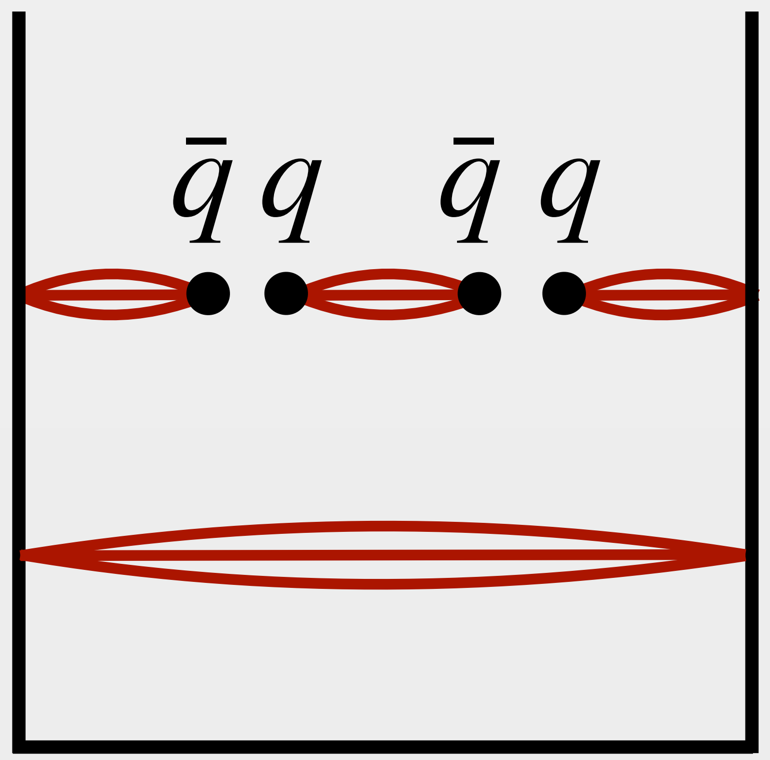

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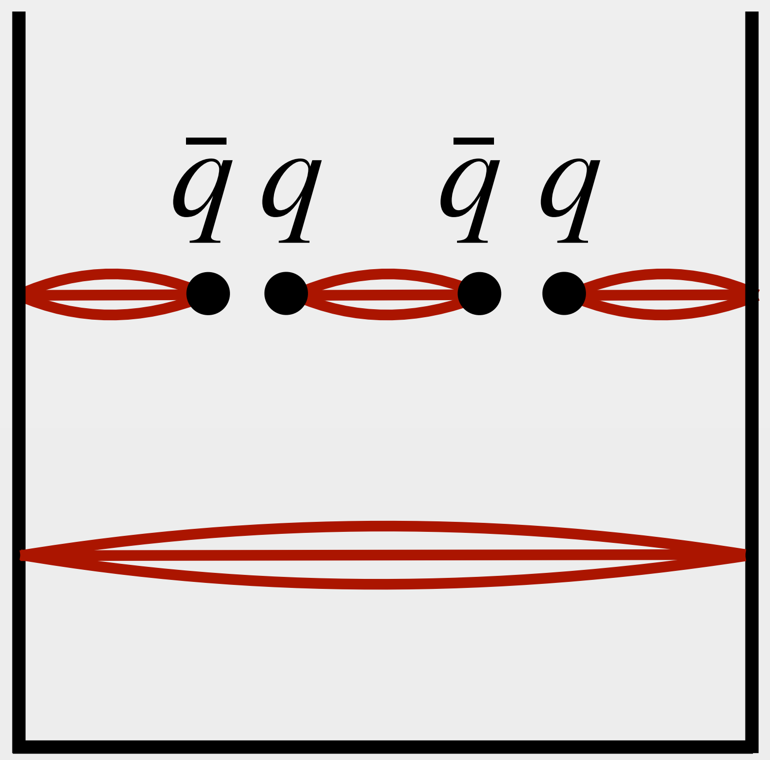

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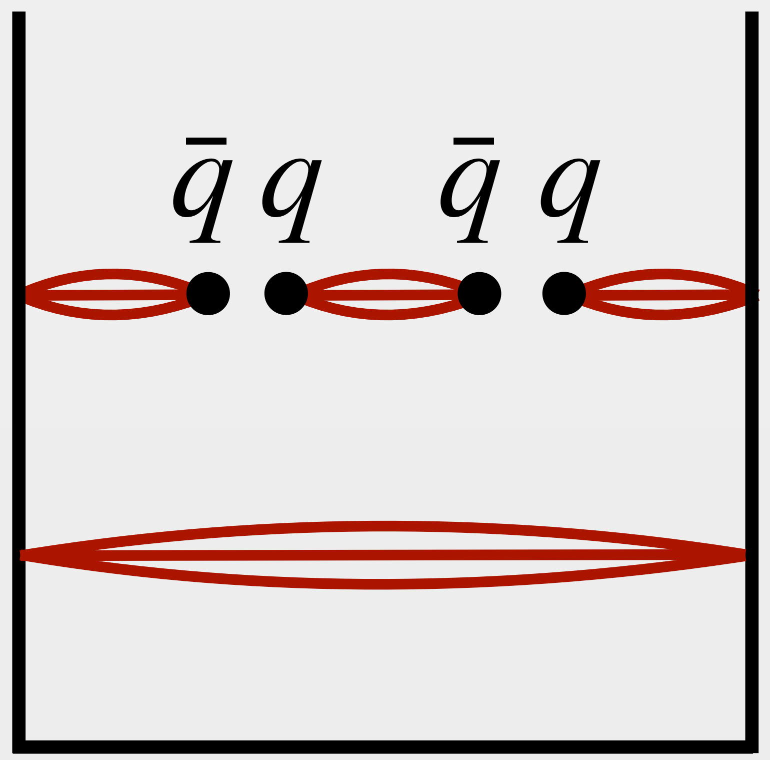

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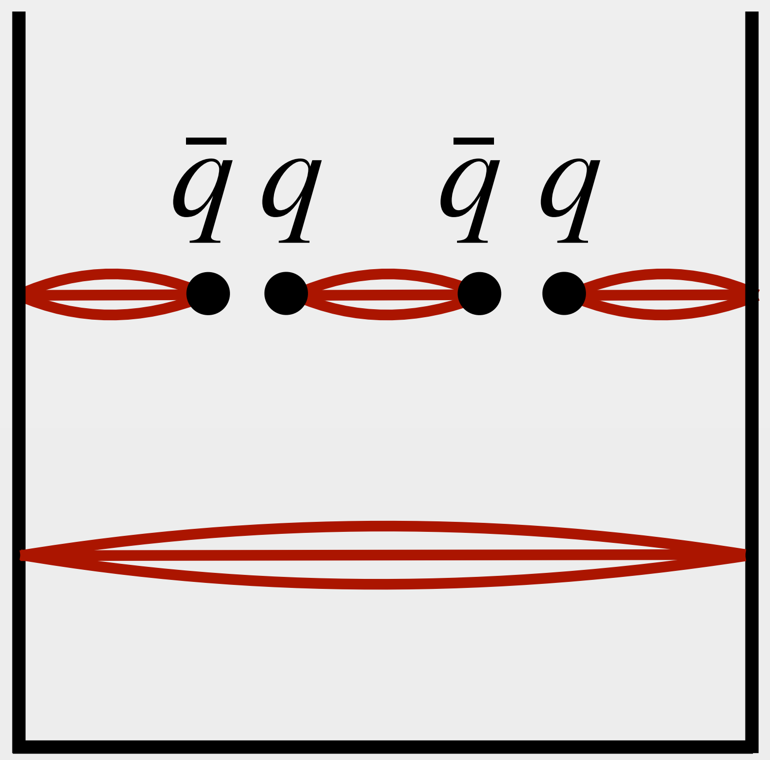

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unfaithful	faithful
$\tilde{G} = U(N_f)_V$	$G = \tilde{G}/\mathbb{Z}_N$
$q_i \to U_{ij}\, q_j$	$q_i \to e^{\frac{2\pi i}{N}} q_i \in SU(N)$ gauge symmetry

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quarks	all gauge invariant operators
$\tilde{G}$ reps & fractional baryon charge $B = 1/N$	G reps & integer baryon charge $B \in \mathbb{Z}$

NOT gauge invariant

meson operator	$\bar{q}_i^a q_j^a$	$B = 0$
baryon operator	$\epsilon_{a_1 \cdots a_N} \bar{q}_{i_1}^{a_1} \cdots q_{i_N}^{a_N}$	$B = 1$

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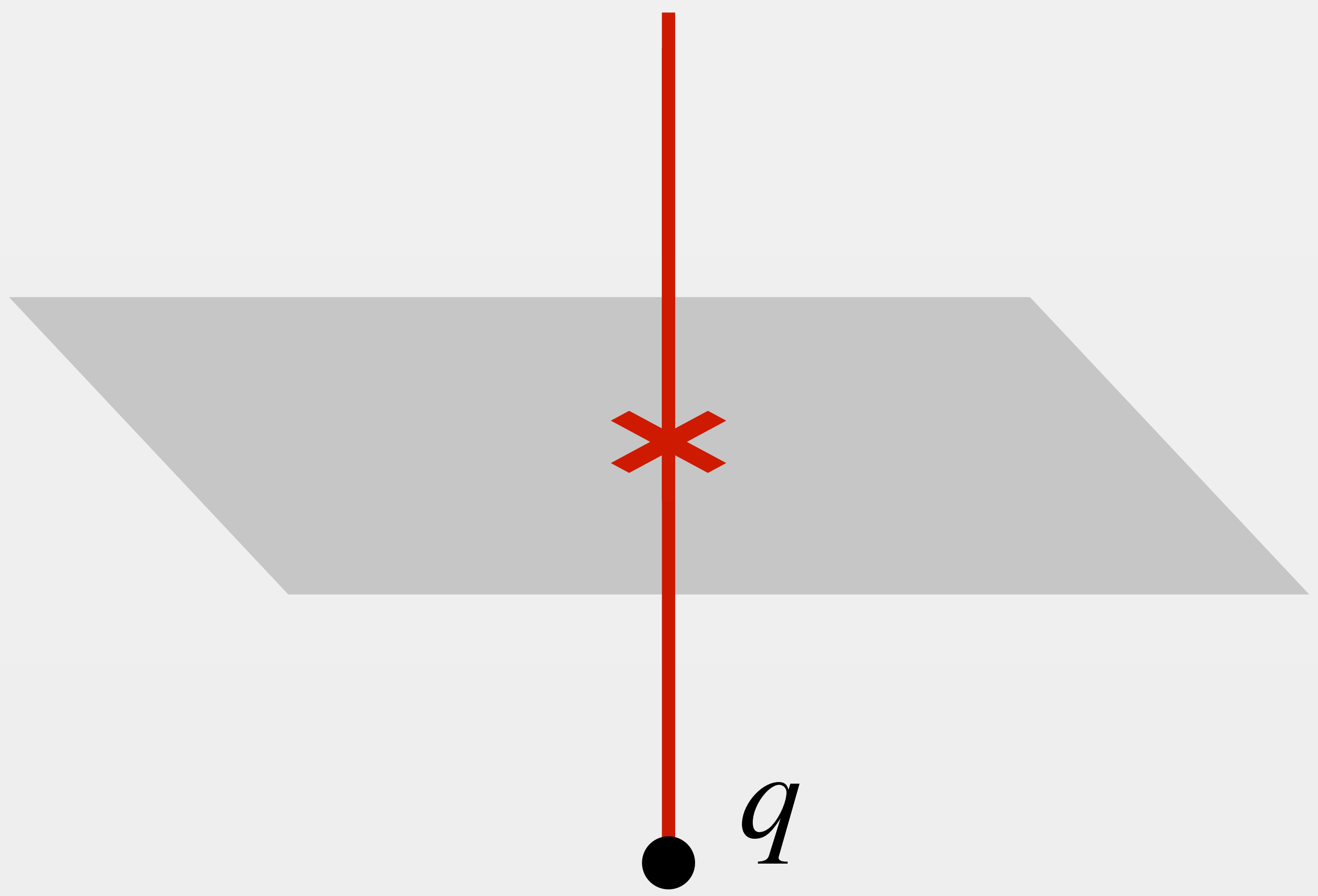

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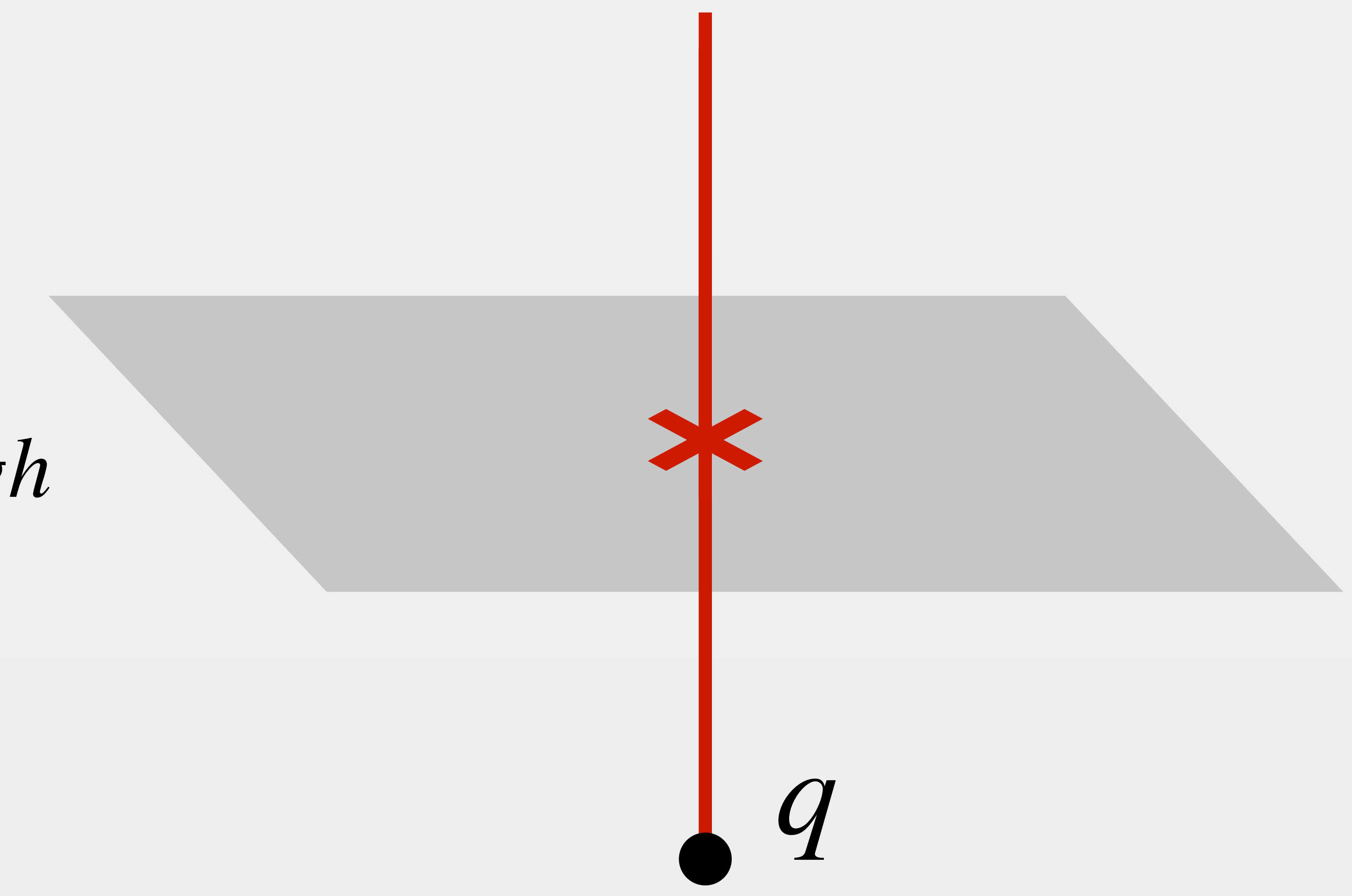

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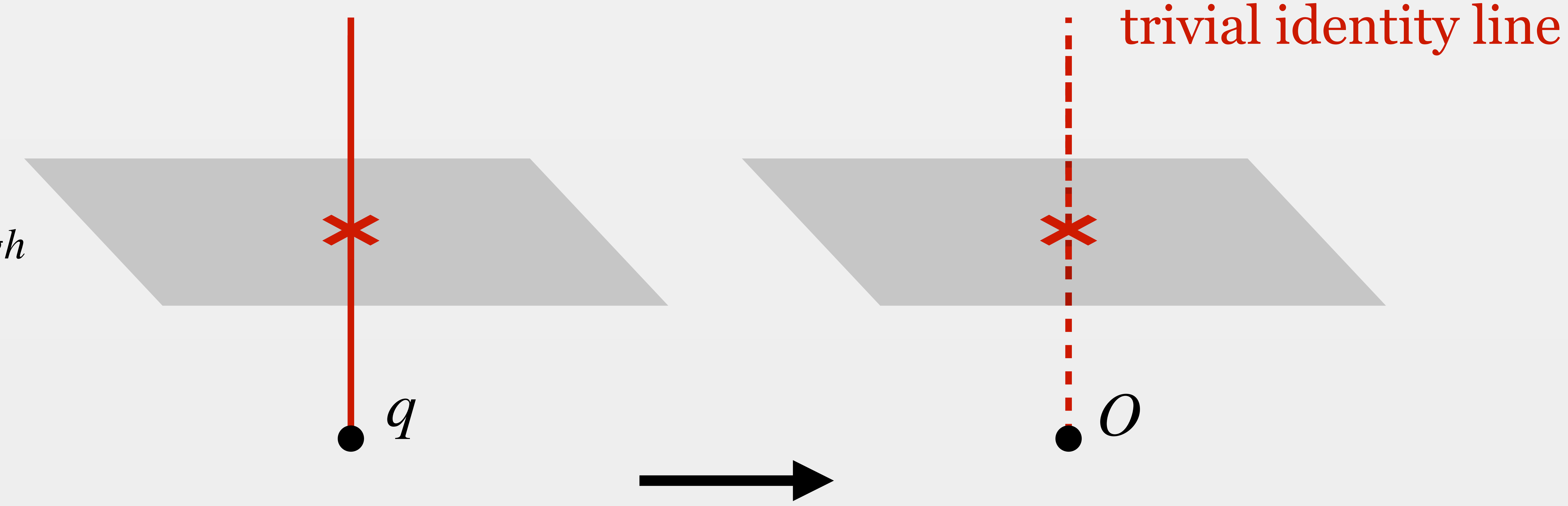

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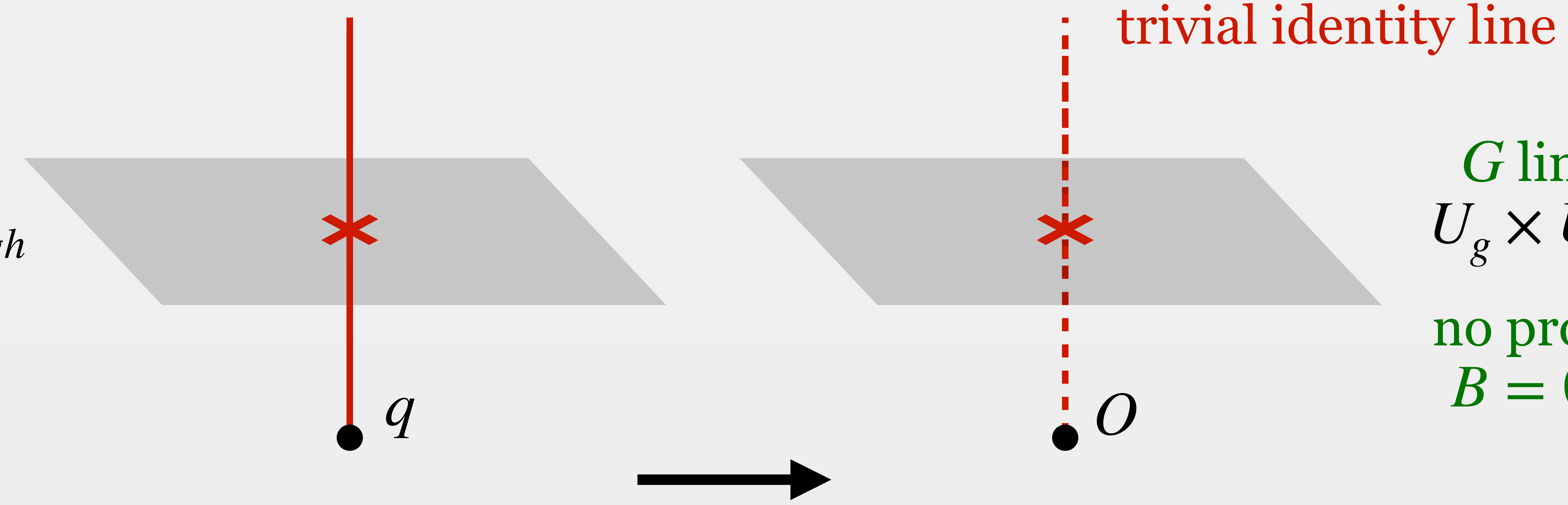

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Strategy: go after $U(1)_B$ symmetry

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finite energy @ infinity $\lim_{r\to\infty} F(r) = 0$

finite energy @ origin $\lim_{r\to 0} F(r) = n\pi$

Baryons as IR Skyrmions

Baryons in QCD are realized as skyrmions in the IR Chiral Lagrangian.

Skyrmions are topologically non-trivial configurations on constant time slices

$$\text{finite energy} \rightarrow \lim_{r\to\infty} U(r,\theta,\phi) = \text{Const}$$

U can effectively be viewed as a map $S^3 \rightarrow SU(N_f)$

topologically distinct configurations are classified by $\pi_3(SU(N_f)) = \mathbb{Z}$

$$U_0(\vec{x}) = \begin{pmatrix} e^{iF(r)\vec{\sigma}\cdot\hat{r}} & \\ & \mathbf{1} \end{pmatrix}$$

$$\underbrace{\qquad\qquad}_{SU(2)} \quad \underbrace{\quad}_{SU(N_f-2)}$$

finite energy @ infinity $\lim_{r\to\infty} F(r) = 0$

finite energy @ origin $\lim_{r\to 0} F(r) = n\pi$

$$B = \frac{1}{24\pi^2}\int \mathrm{Tr}[(U^\dagger dU)^3] = \frac{2F(0) - \sin(2F(0))}{2\pi} = n$$

Wilson Line as Disorder Operator

[HTL to appear]

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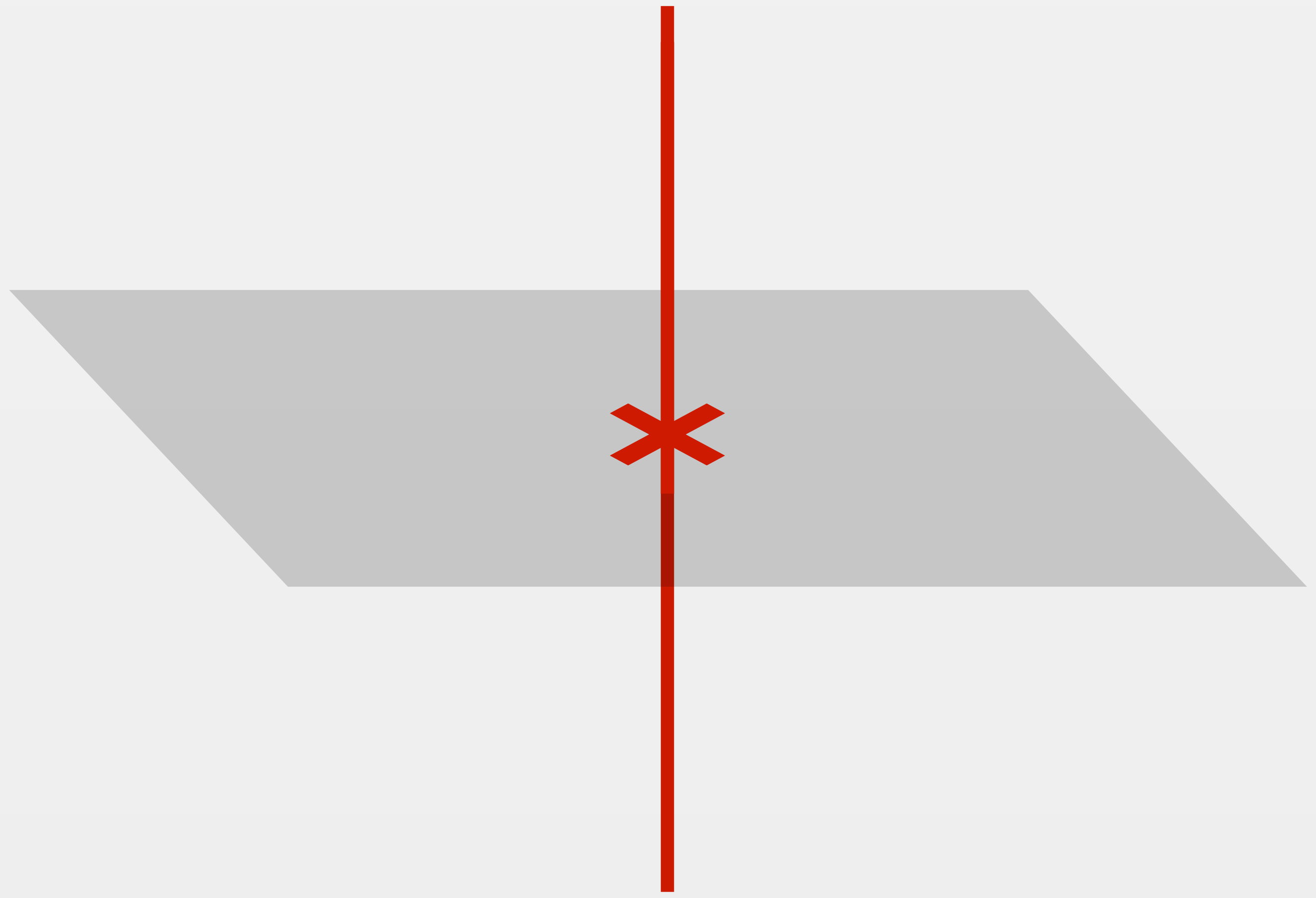
Baryon charge $B = 1/N \bmod 1$ is fractional in the Wilson line defect Hilbert space.



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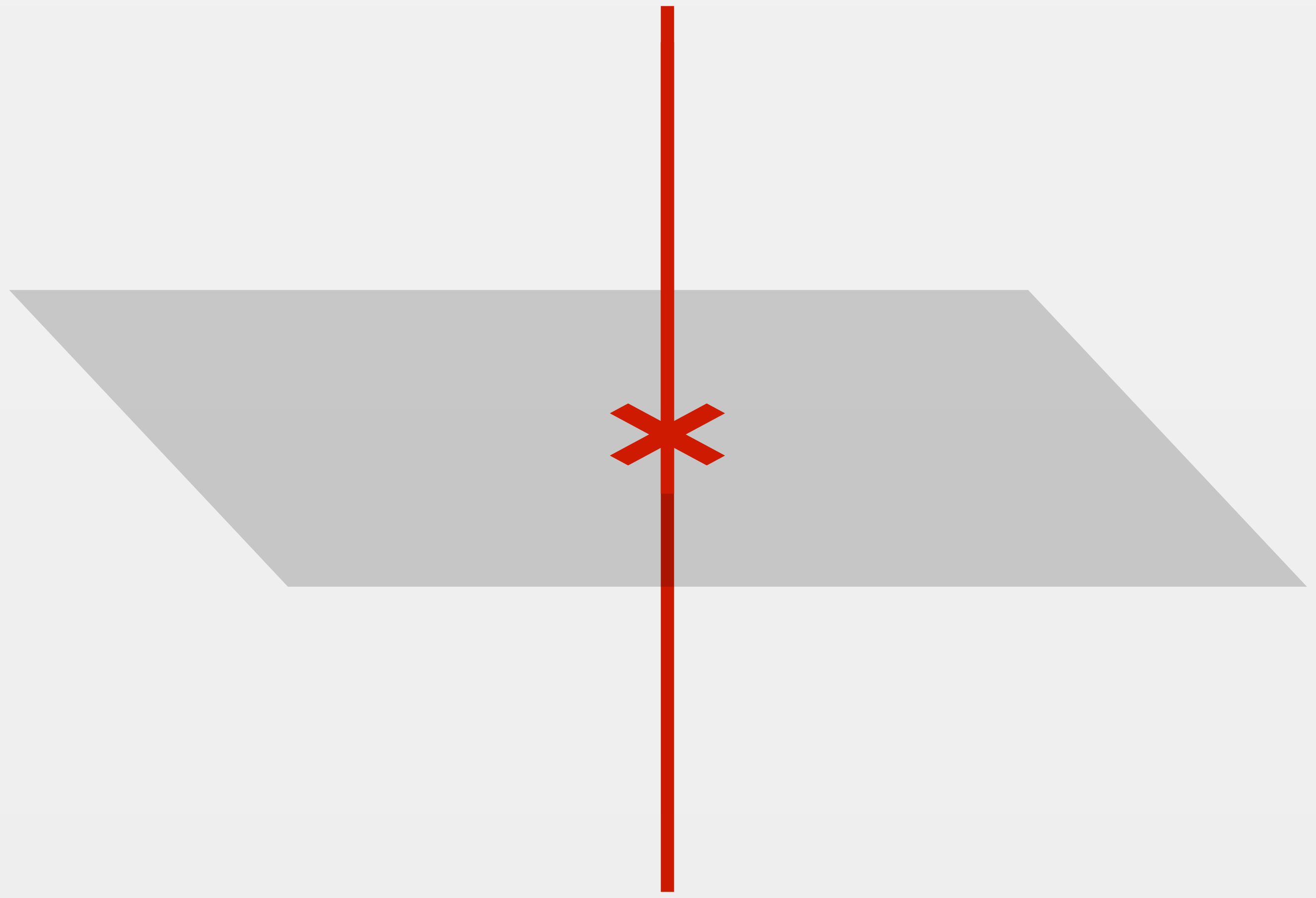

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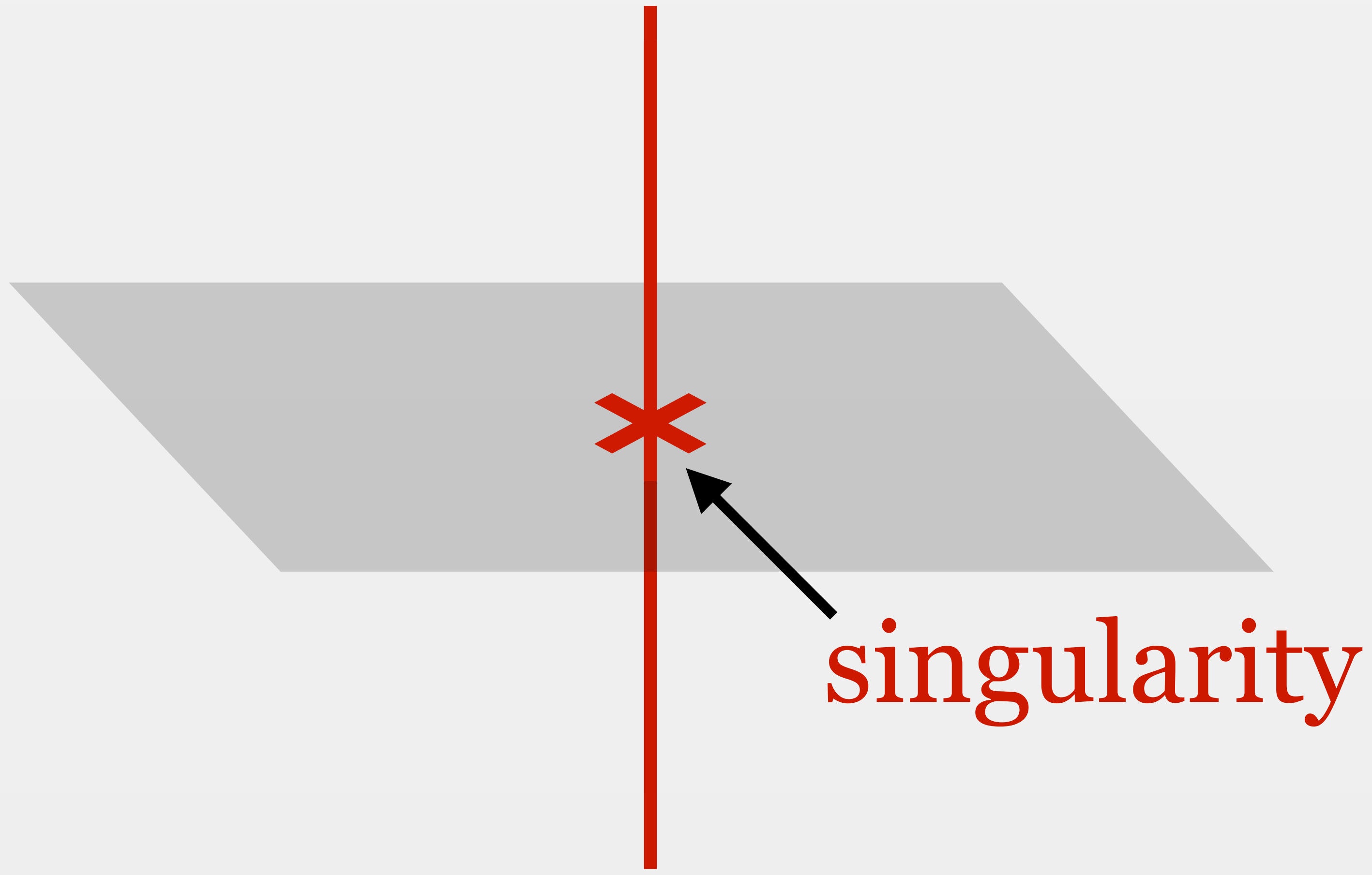

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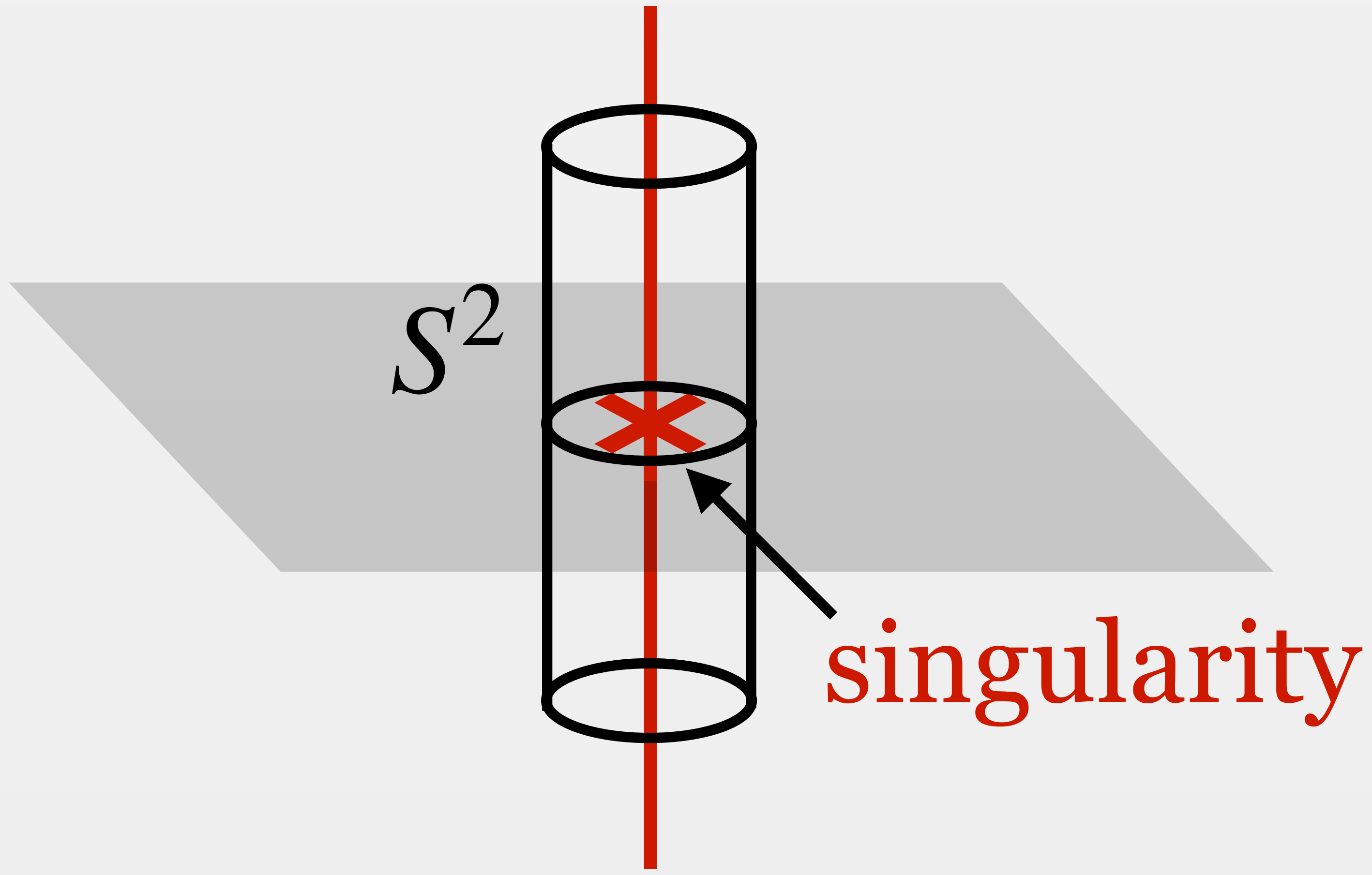

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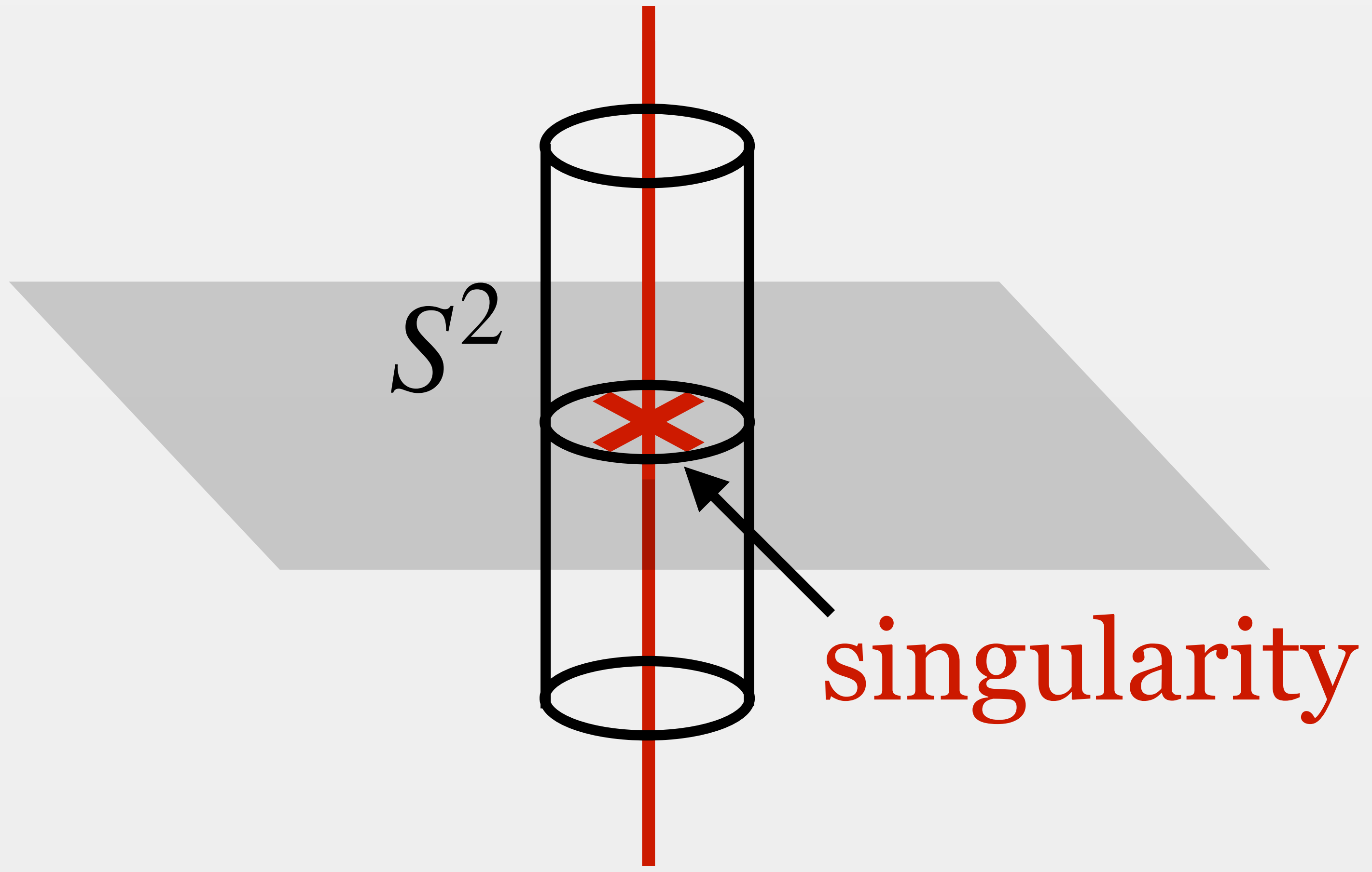

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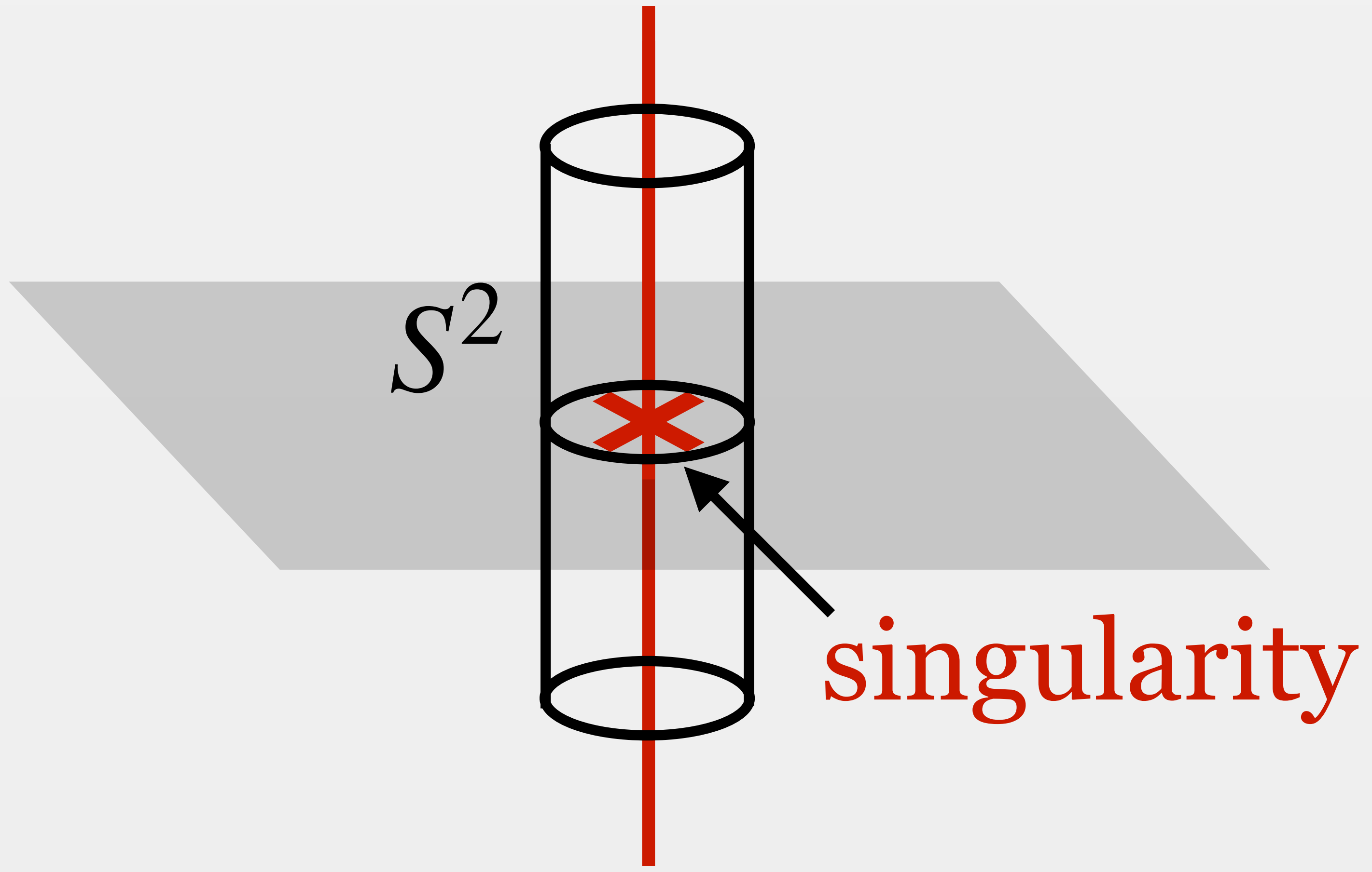

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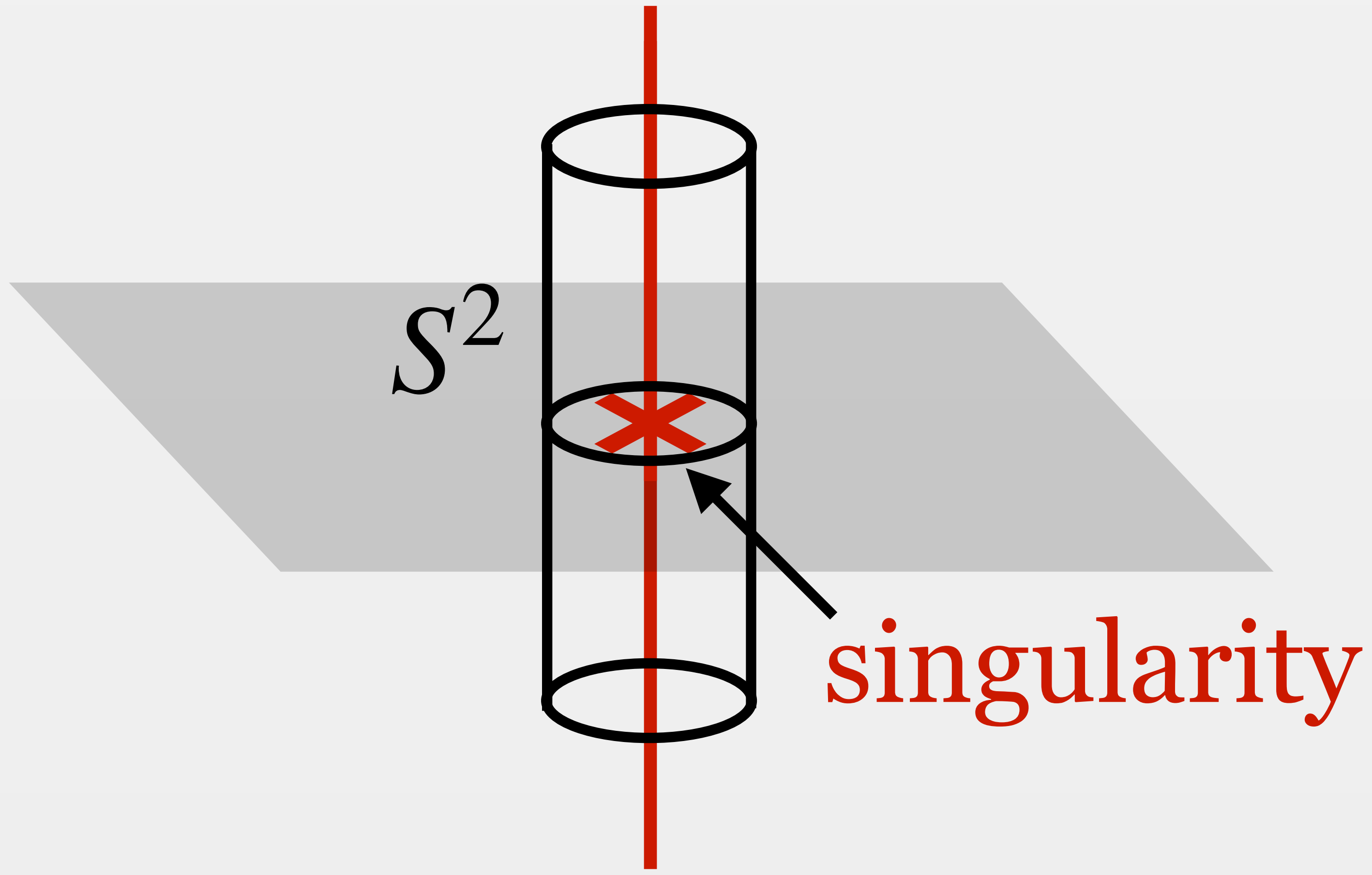

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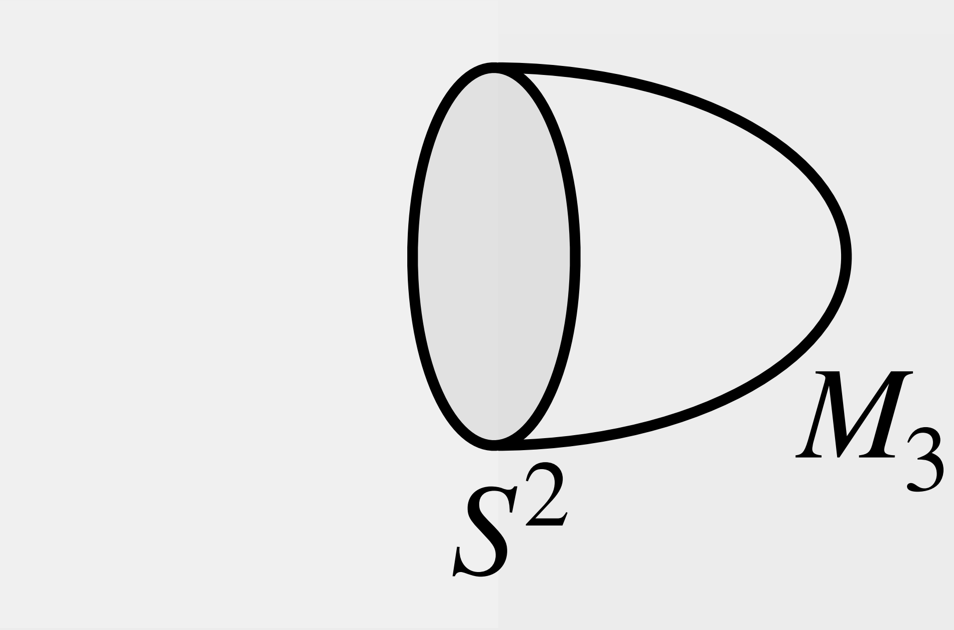

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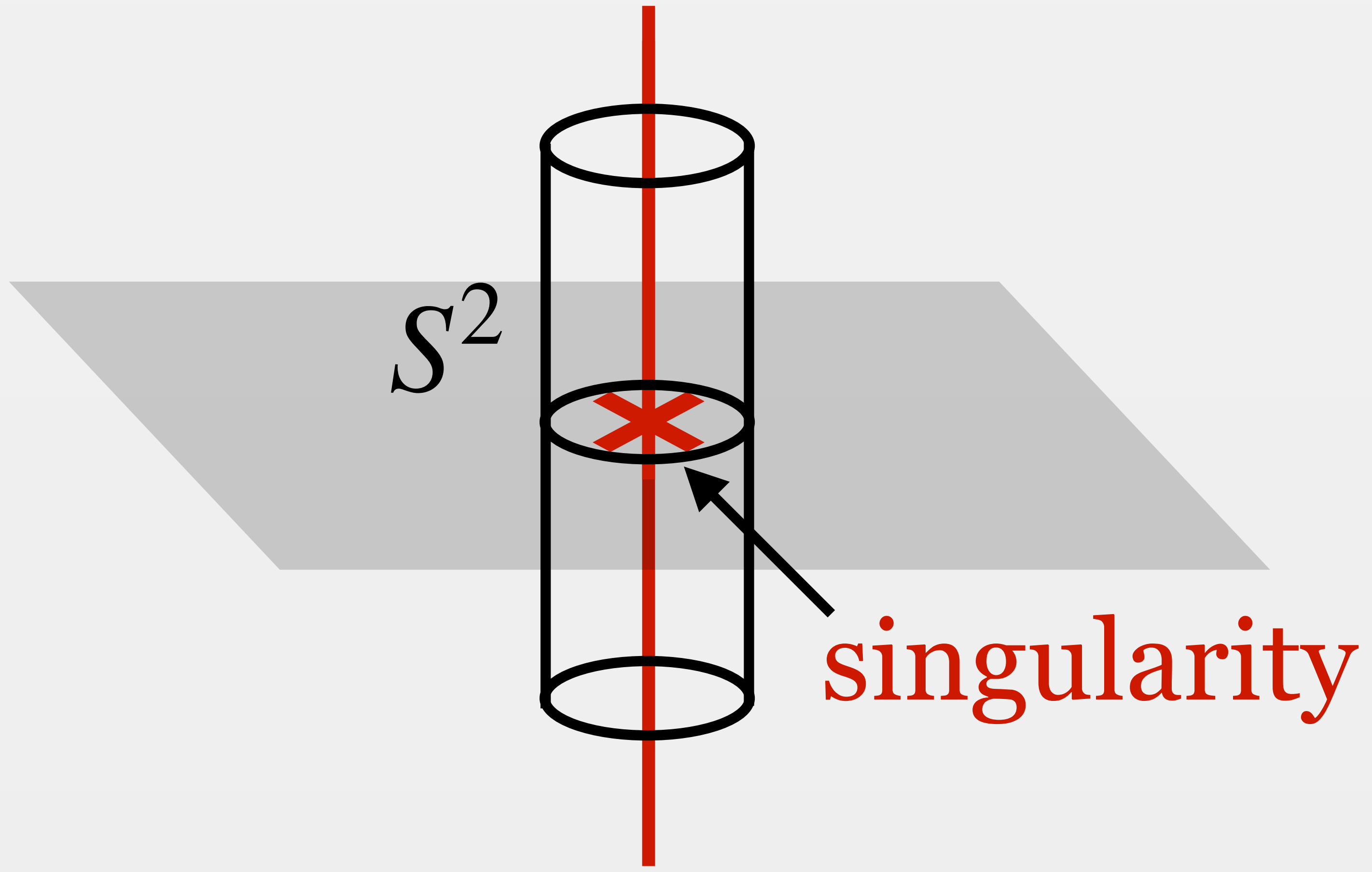

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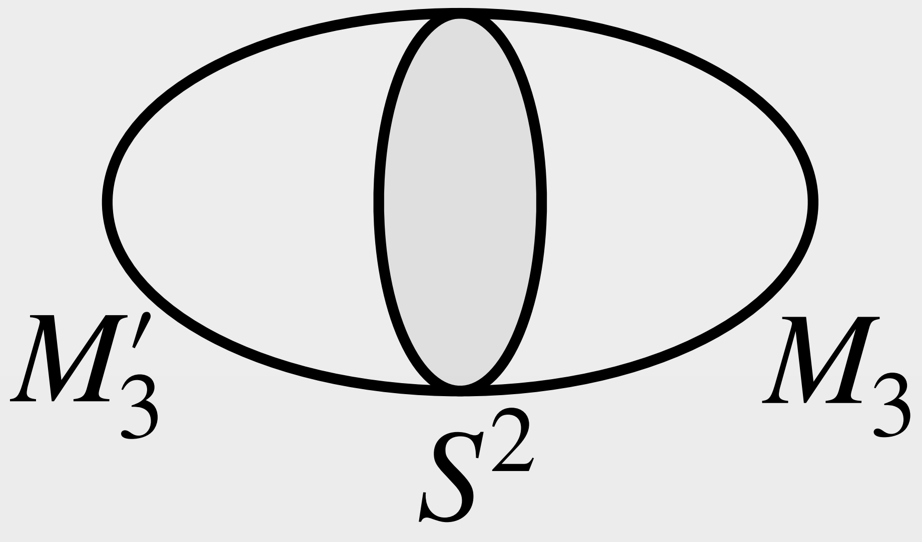

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Singular Fractional Skyrmions

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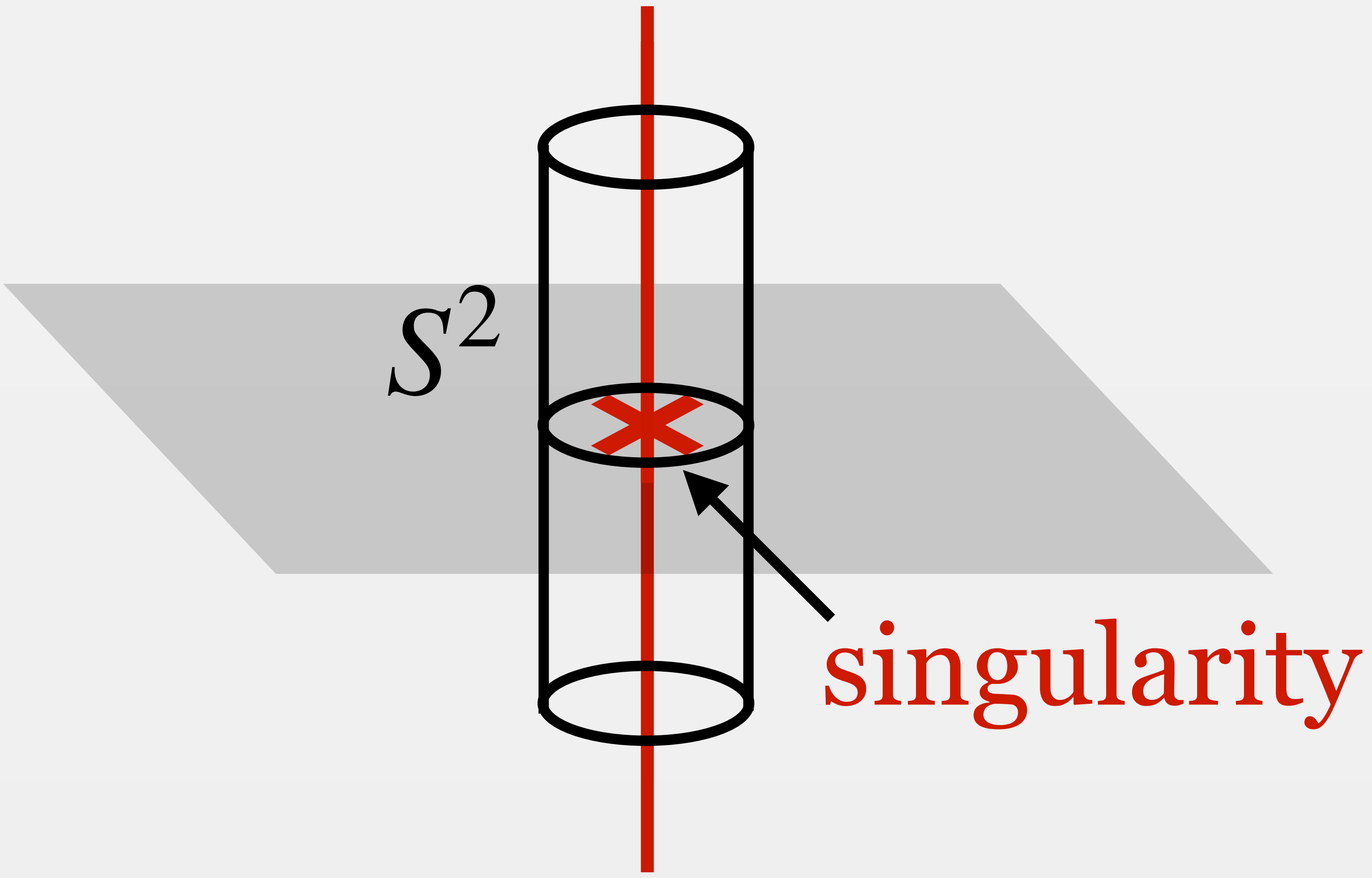

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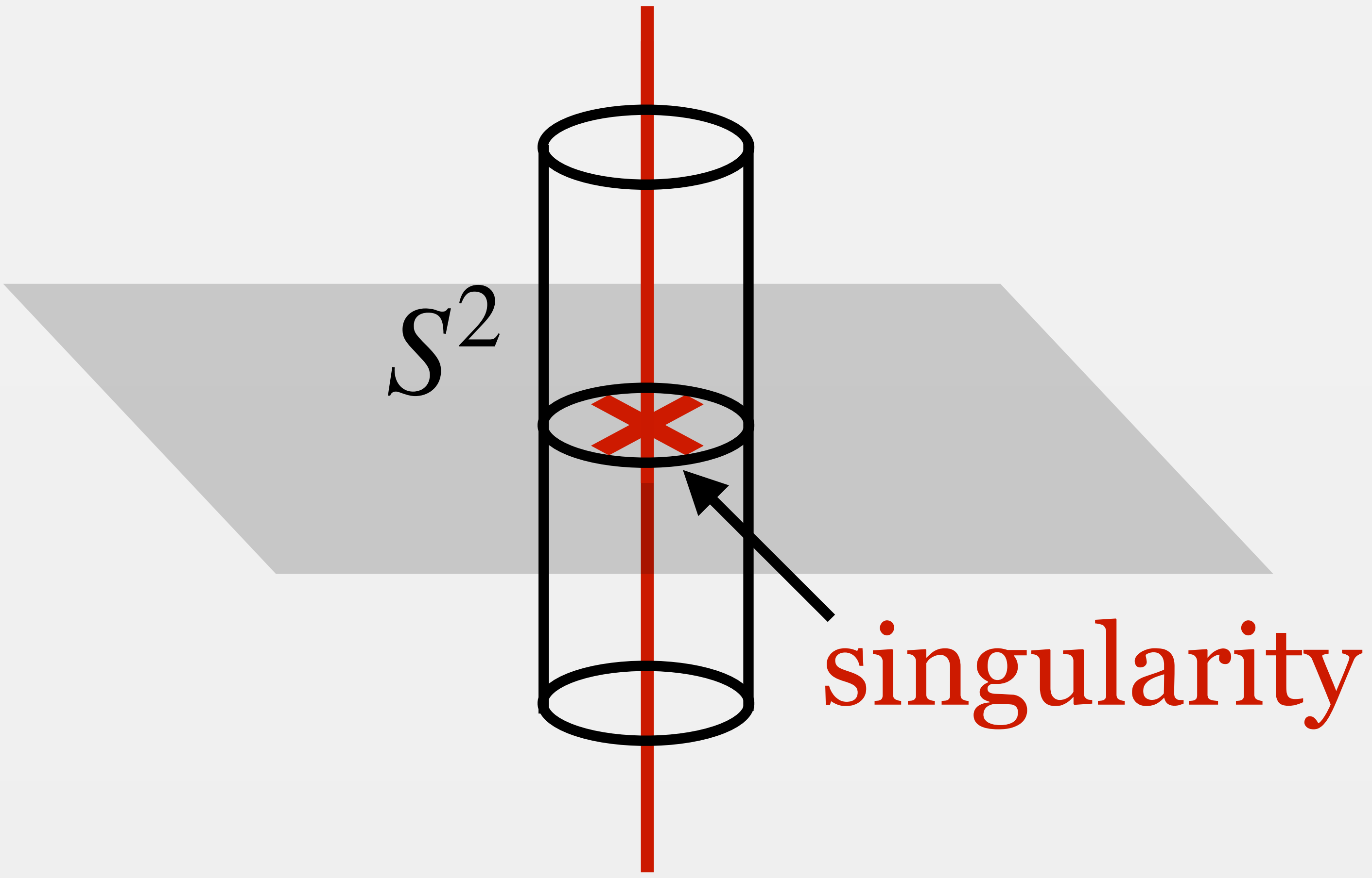

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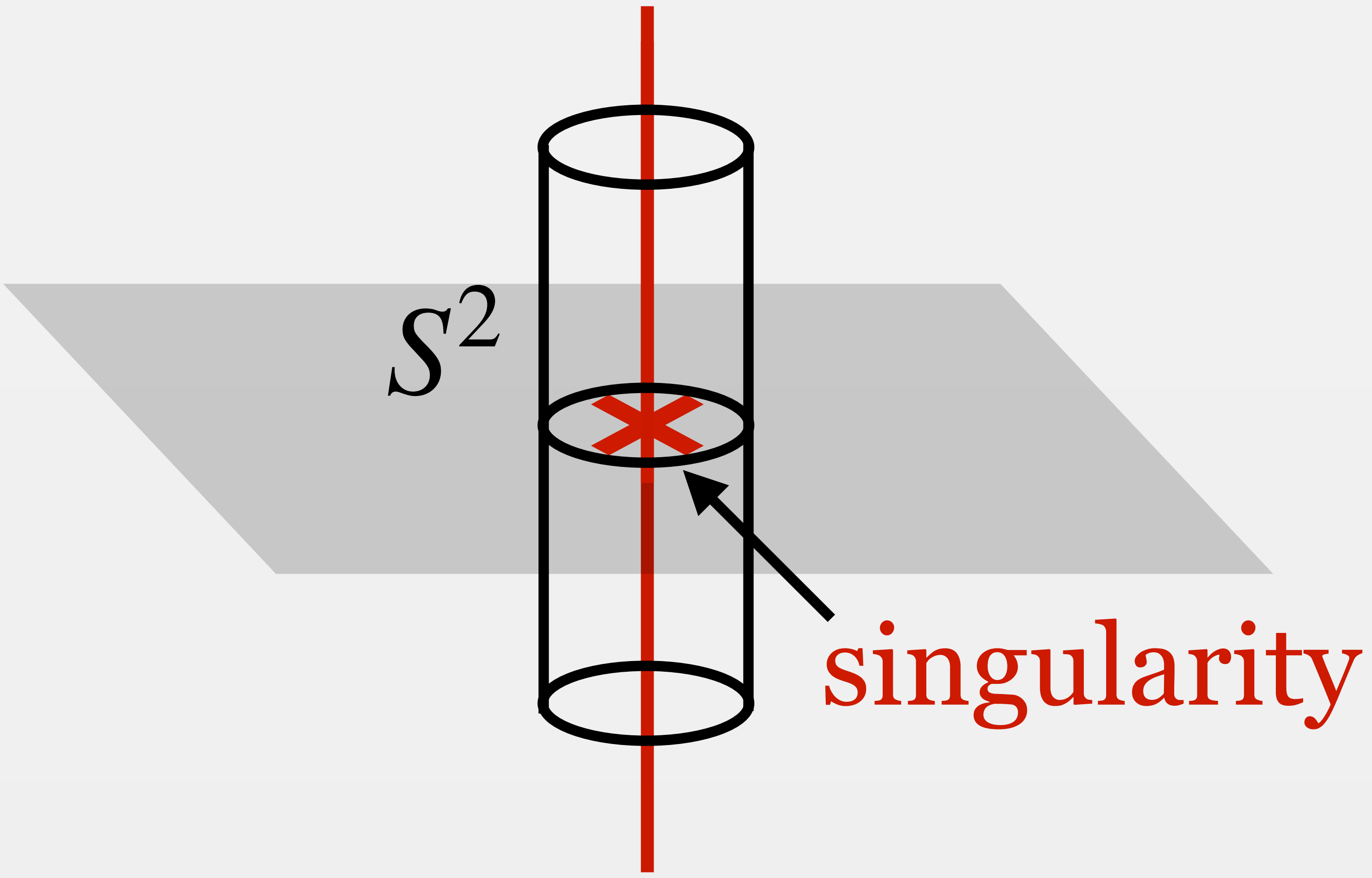

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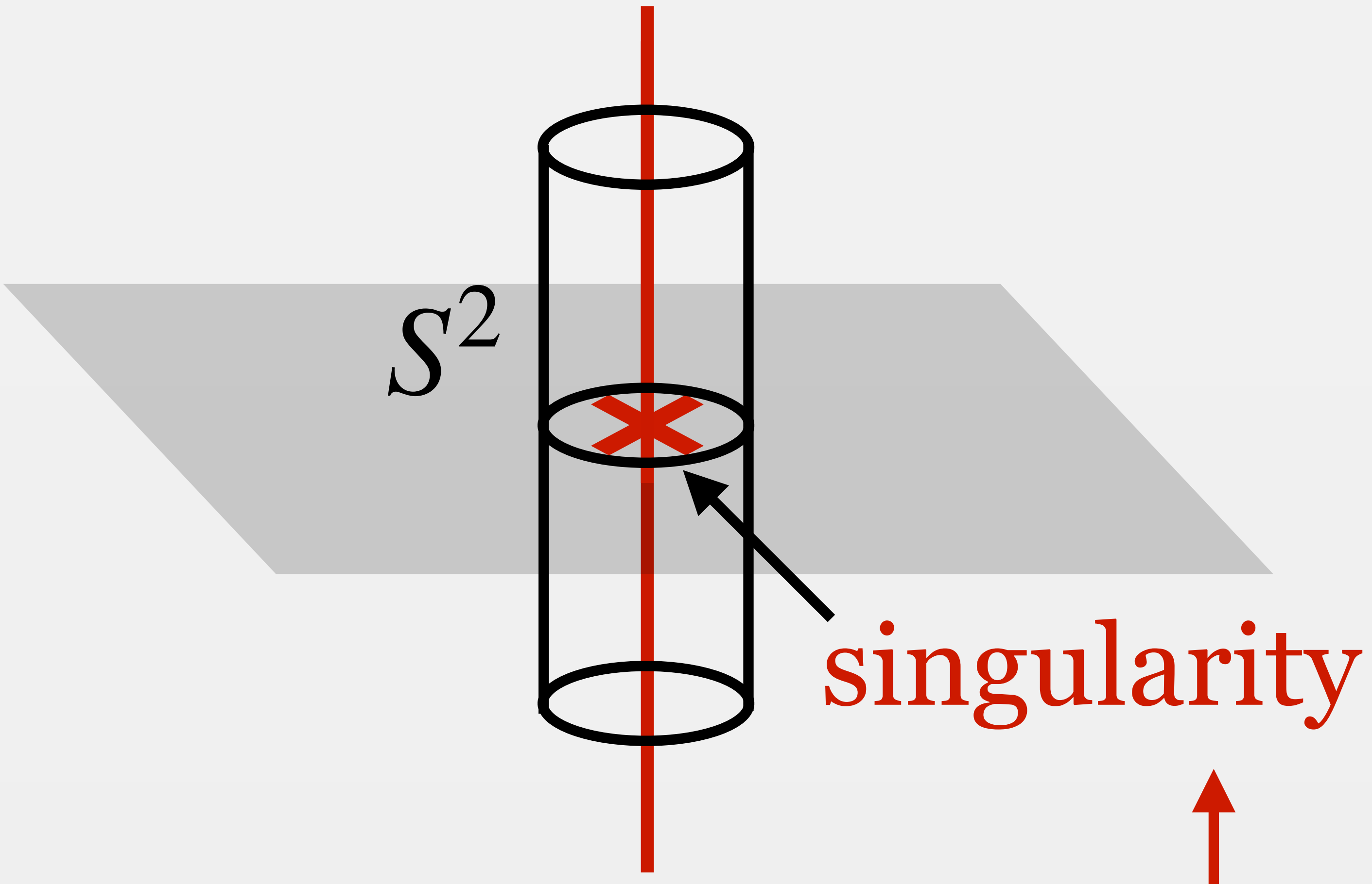

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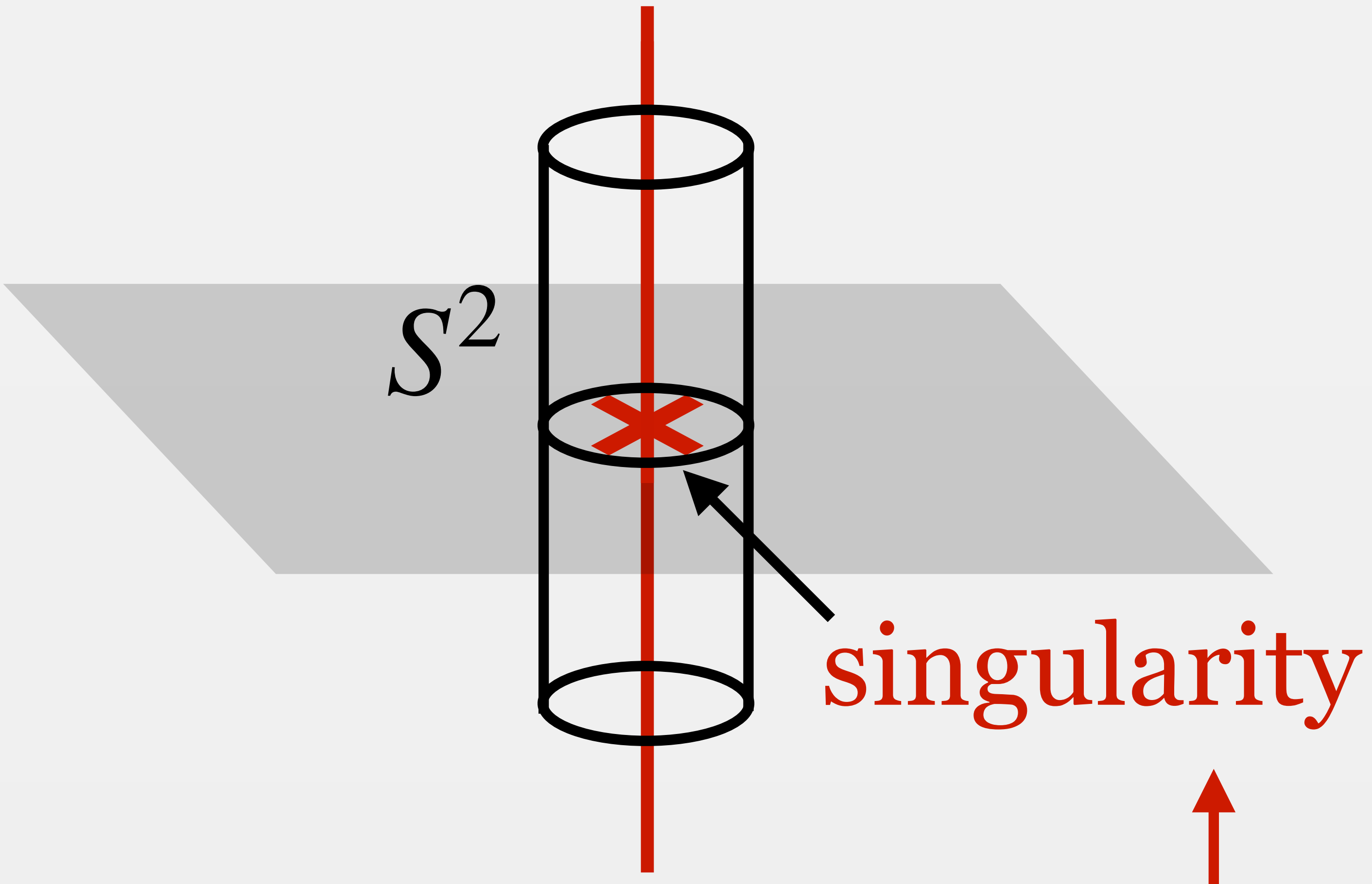

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Add rotation by introducing collective coordinate

Boundary value: $U(t, \vec{x}) = A(t)U_0(\vec{x})A(t)^\dagger$

[HTL to appear]

Why is the fractional skyrmion number quantized in the IR?

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The fractional skyrmion has no well-defined statistics unless $B = n/N$

Statistics of Skrymions

[..., Witten '83, ...]

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Let us first recap how to detect the statistics of a smooth skyrmion.

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Fermionic Statistics of Fractional Skyrmions

[HTL to appear]

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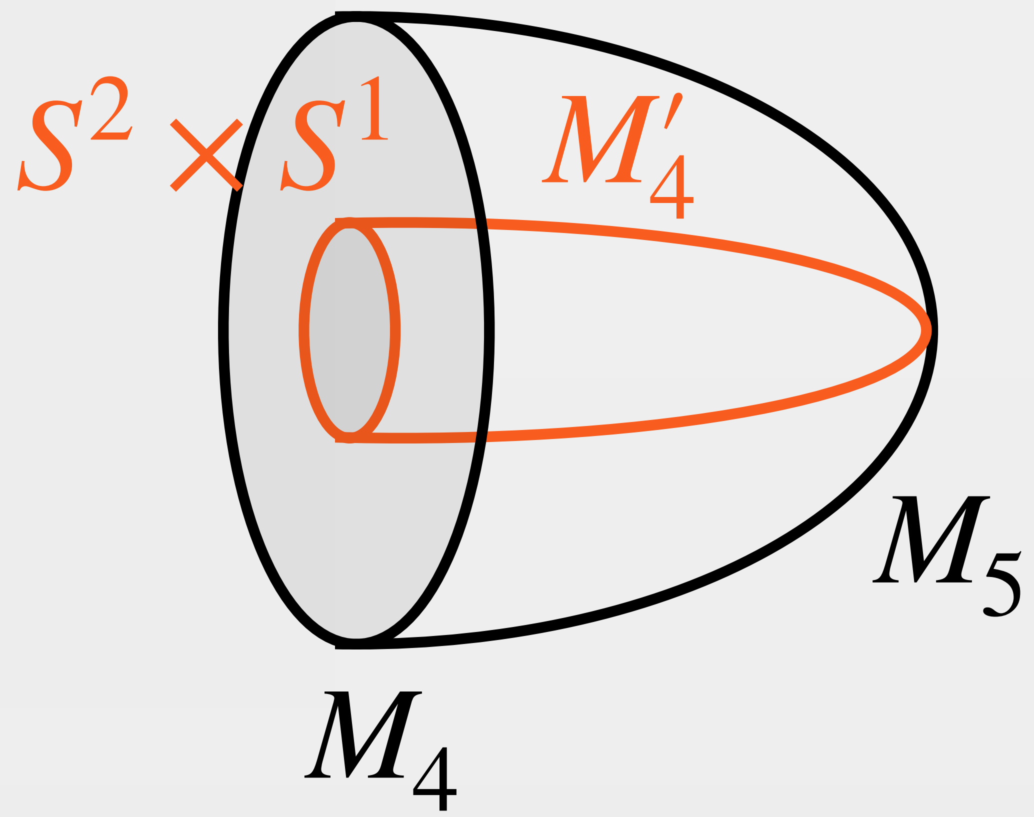

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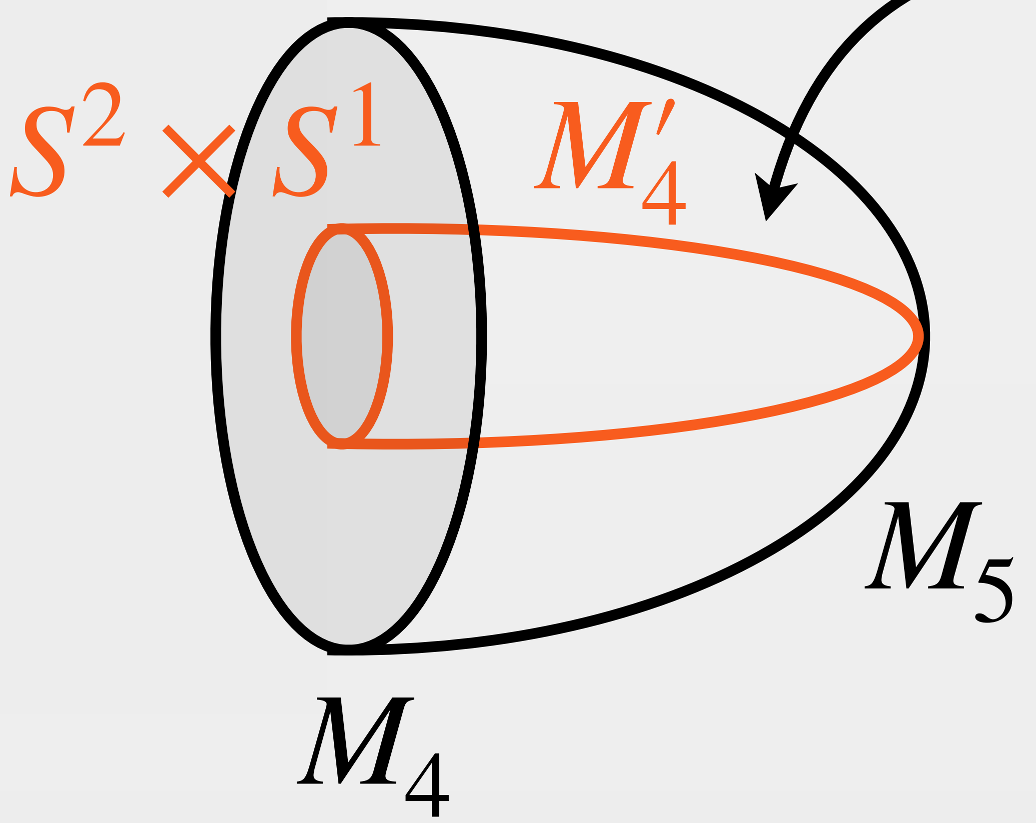

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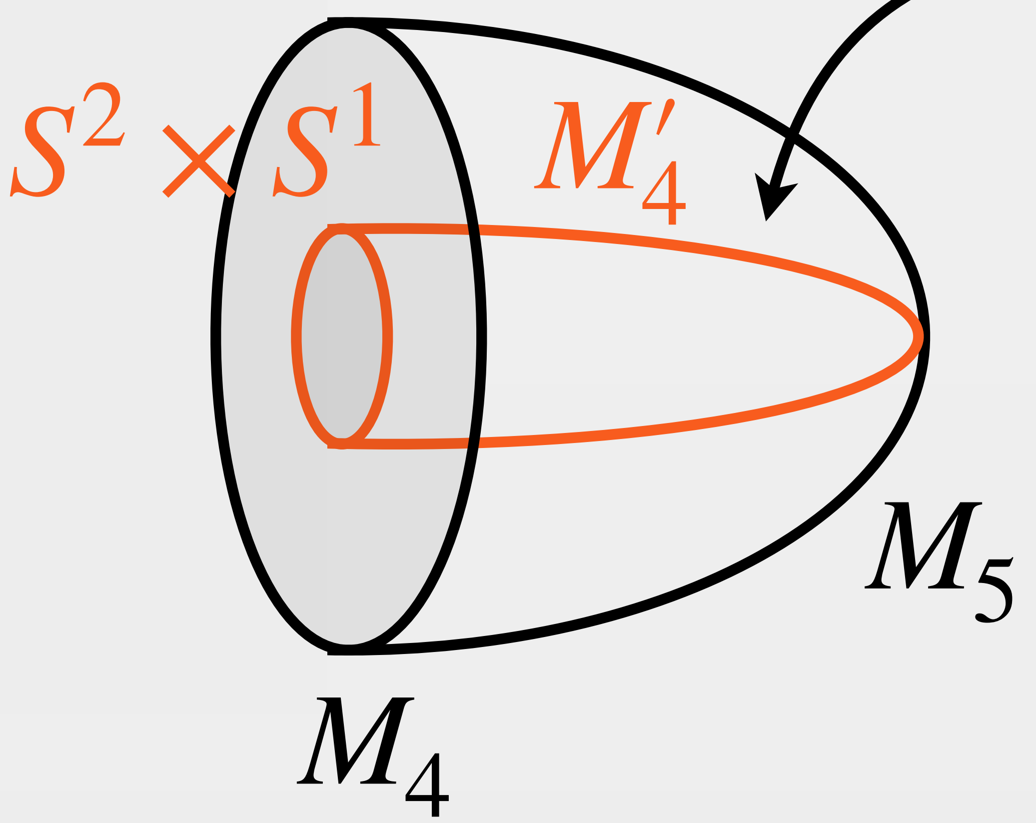

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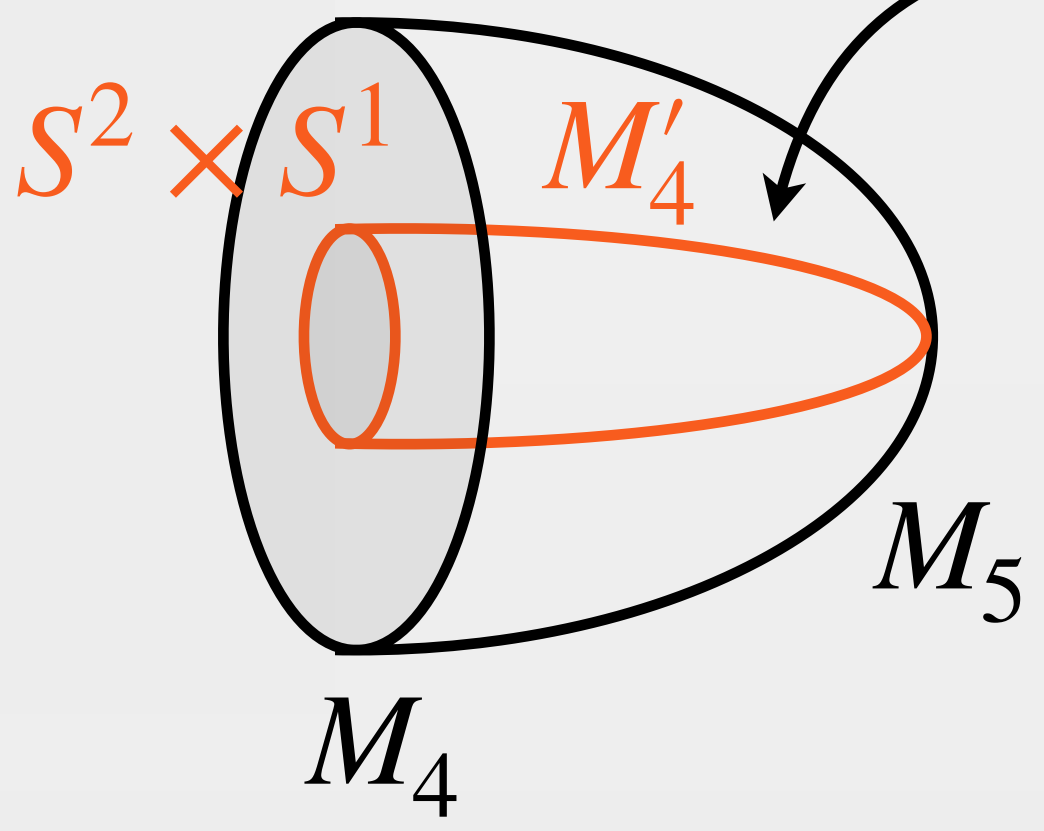

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$$B = 1/N \rightarrow e^{iNS_{WZ} + iNS_{BWZ}} = -1$$

fermionic statistics of quark

[HTL to appear]

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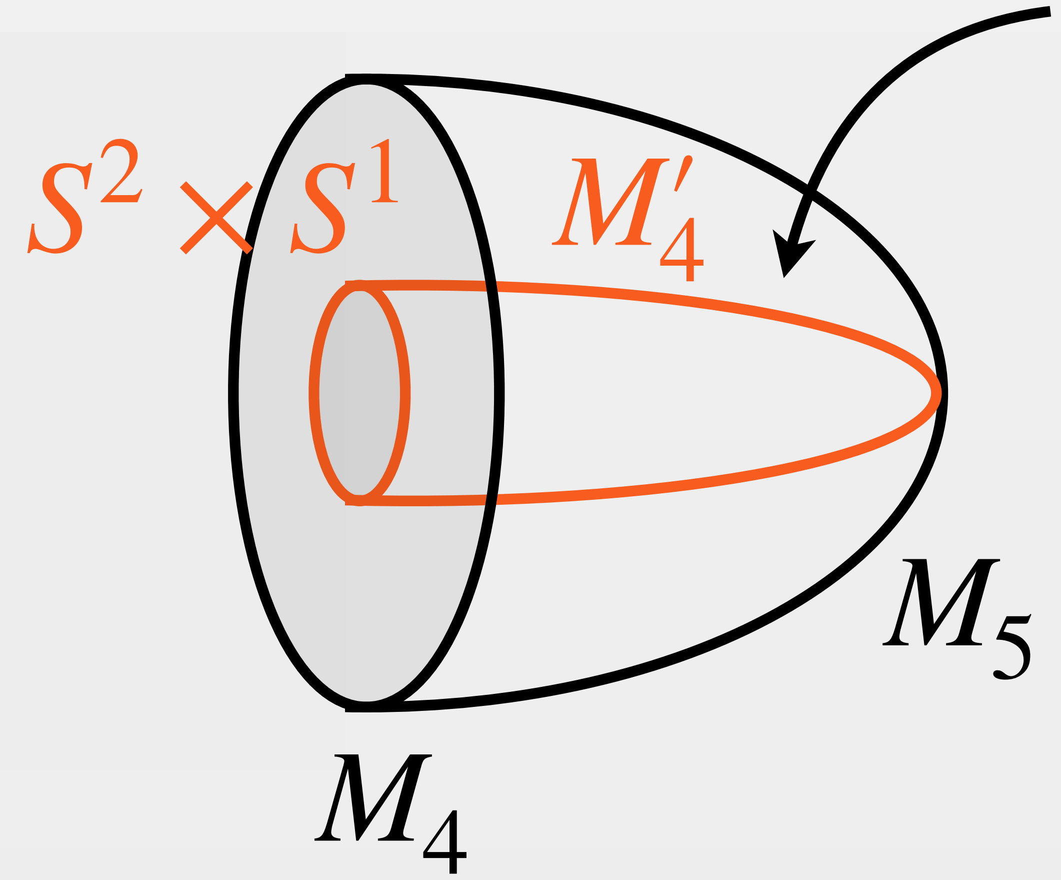

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Fractional skyrmions have well-defined statistics only for $B = n/N$

[HTL to appear]

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Heavy-Light Hadrons

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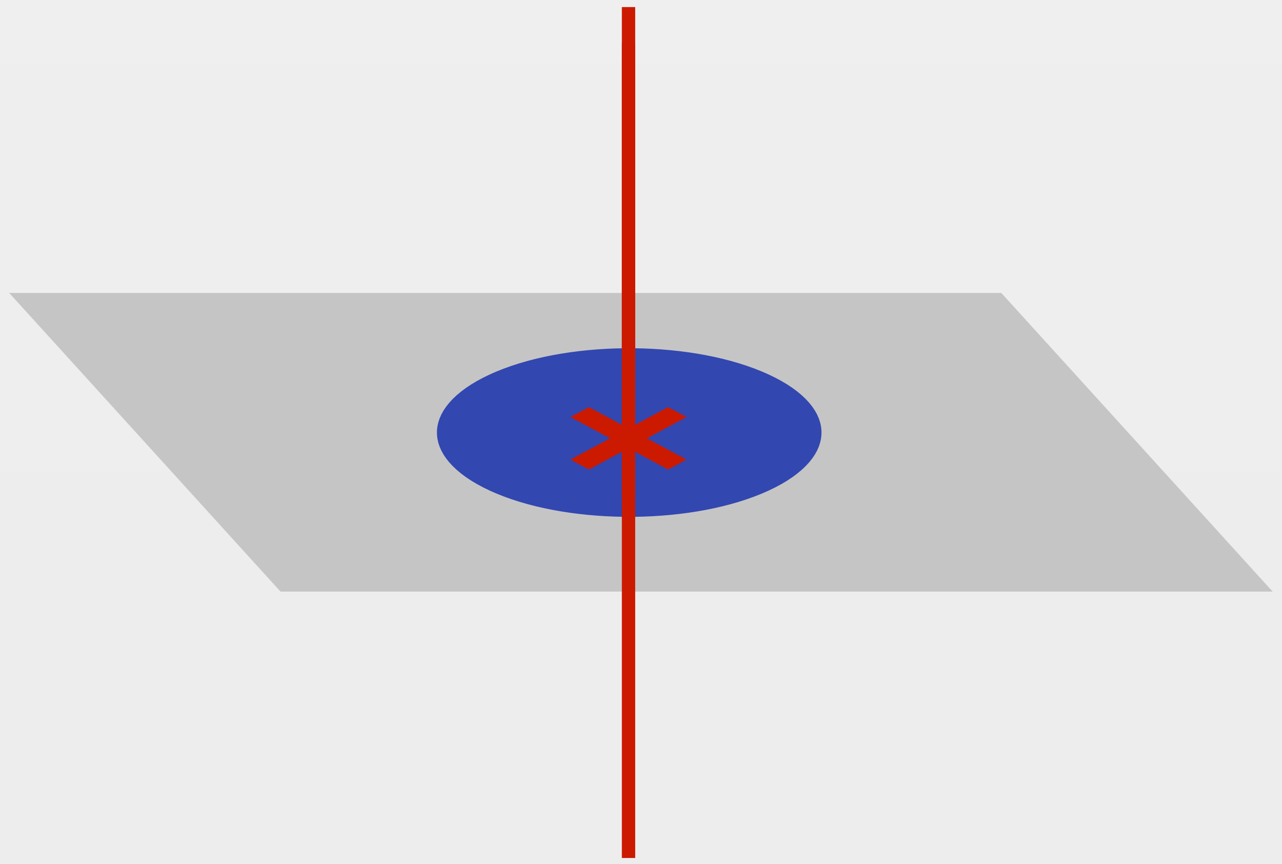

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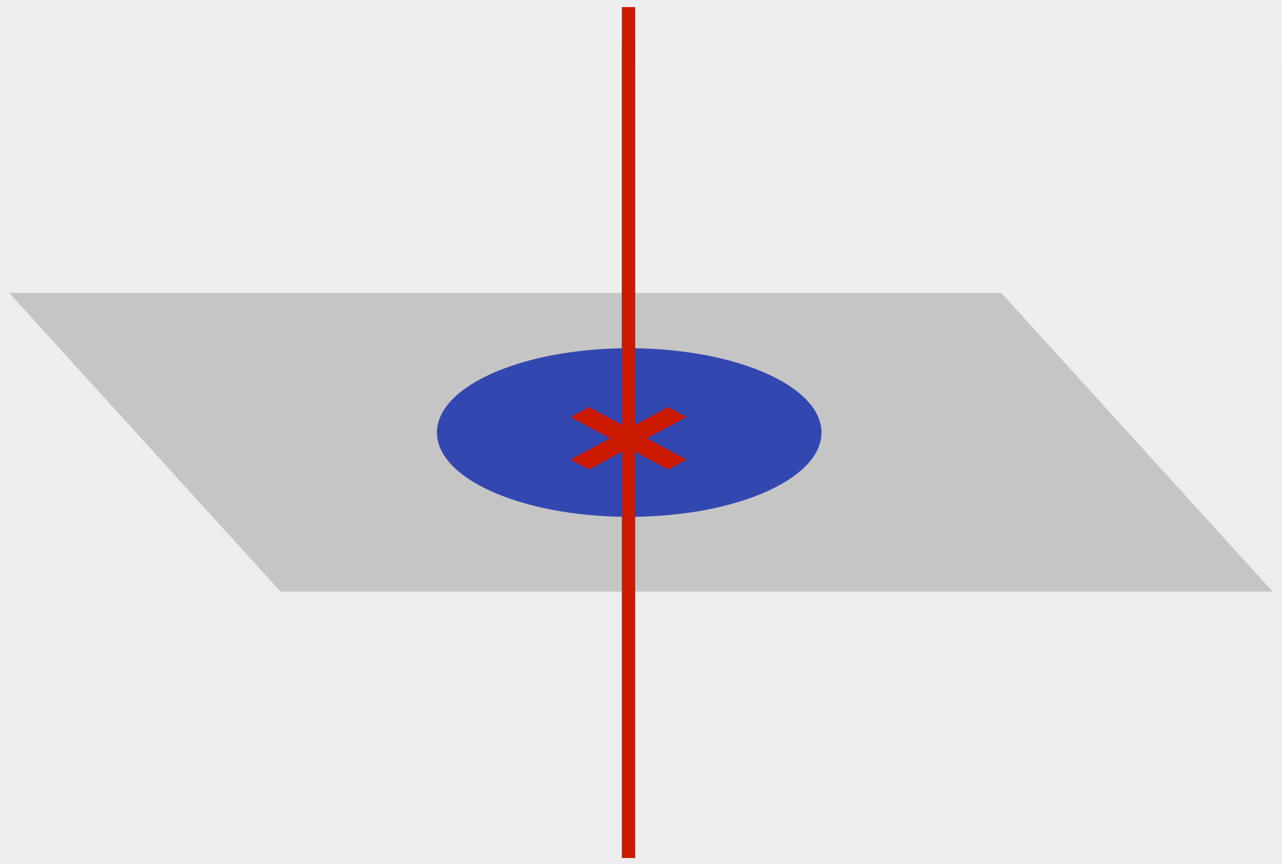
Heavy-Light Hadrons

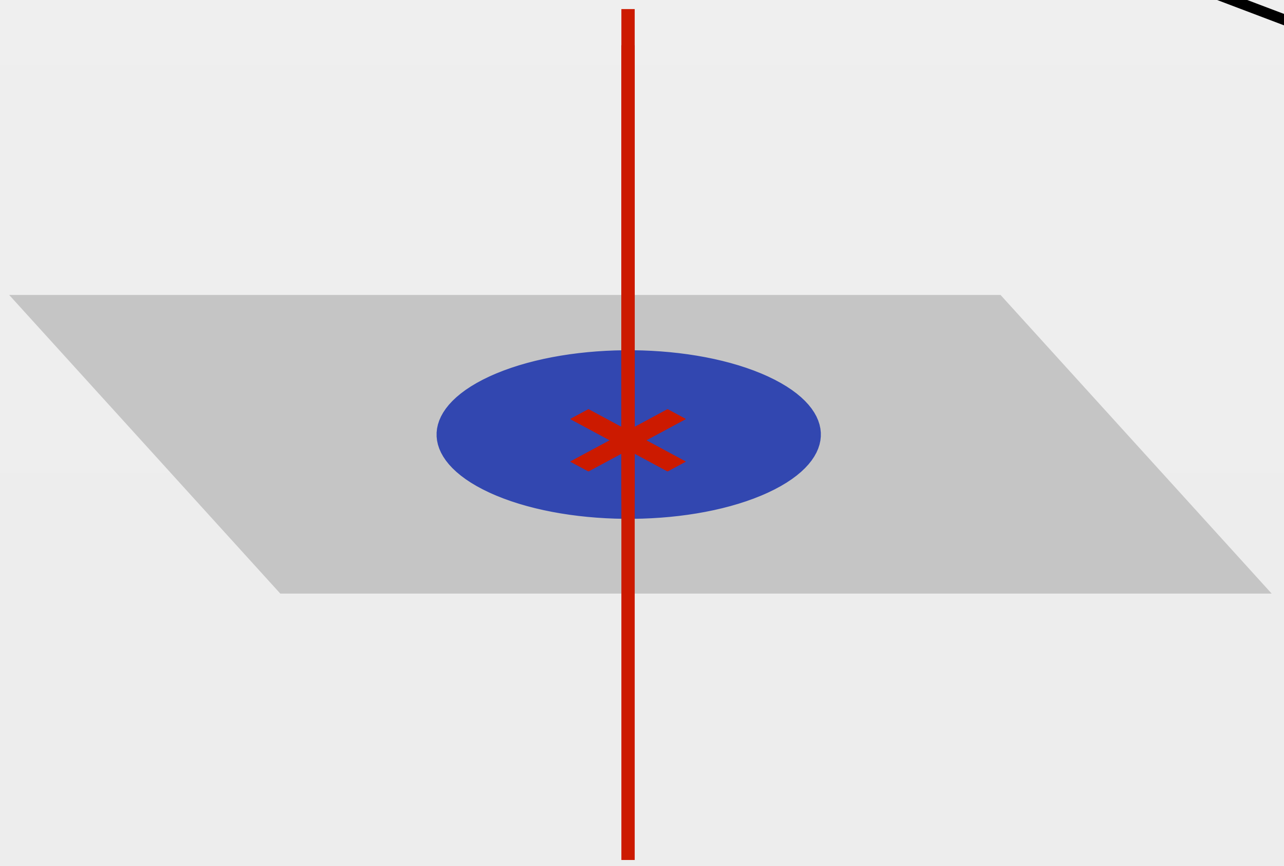
worldline of a heavy quark
Wilson line



What are the physical meaning of these fractional skyrmions?

Heavy-Light Hadrons

worldline of a heavy quark
Wilson line



light quarks + gluons
fractional skyrmion

Real-World QCD

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Heavy quarks: Modeled by Wilson lines

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mass difference between heavy-light hadrons should come from the light degrees of freedom

$SU(2)$ Skyrmion Model

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Let us work with the simplest $SU(2)$ skyrmion model (include up & down quarks)

$$\mathscr{L} = \frac{f_\pi^2}{16}\mathrm{Tr}(\partial^\mu U^\dagger \partial_\mu U) + \frac{1}{32e^2}\mathrm{Tr}[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U]^2$$

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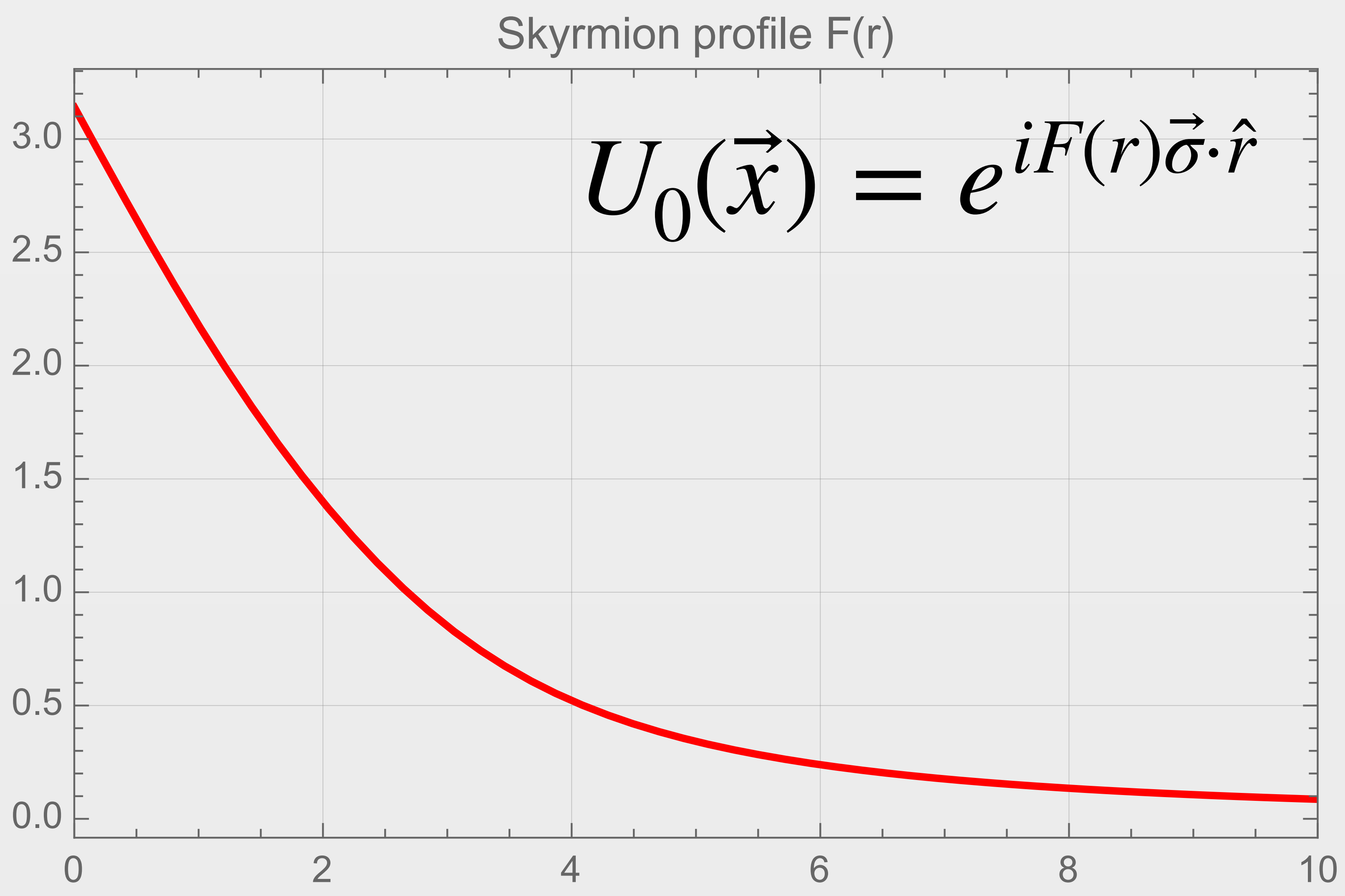

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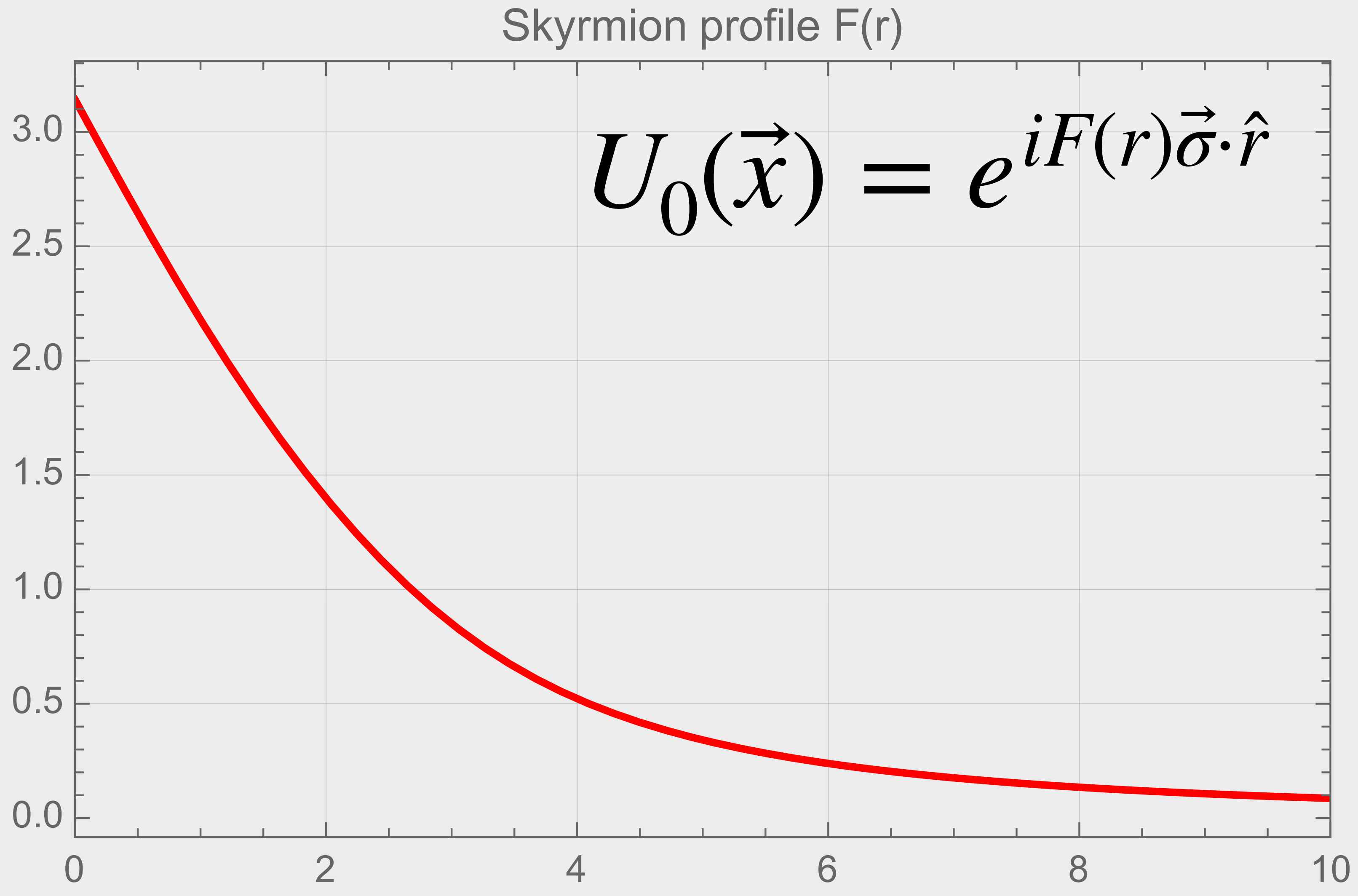

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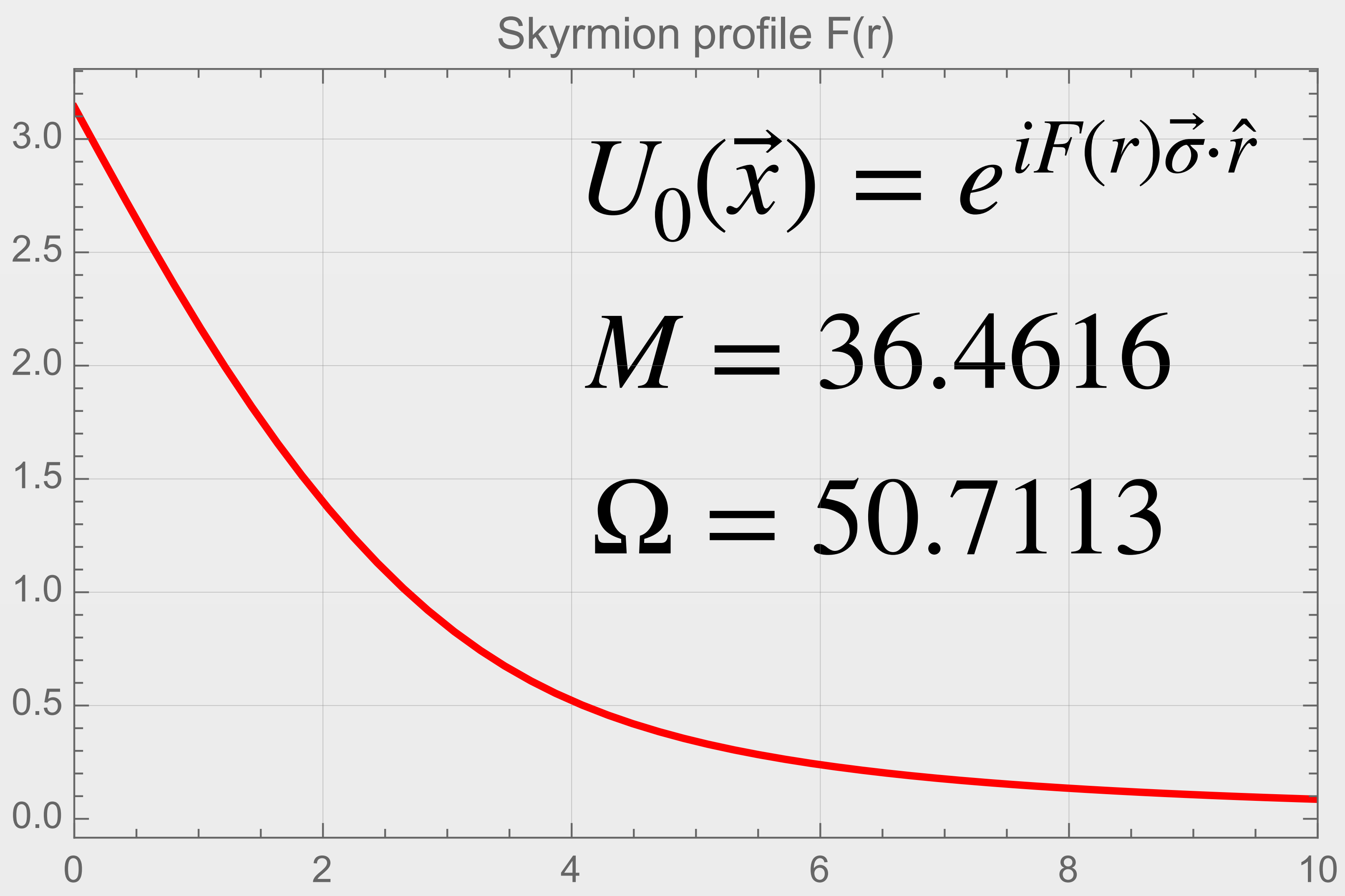

$$U_0(\vec{x}) = e^{iF(r)\vec{\sigma}\cdot\hat{r}}$$

$$M = 36.4616$$

$$\Omega = 50.7113$$

$$m = \frac{f_\pi}{e}M + \frac{3f_\pi e^3}{4\pi\Omega}I(I+1) \qquad I = \text{isospin}$$

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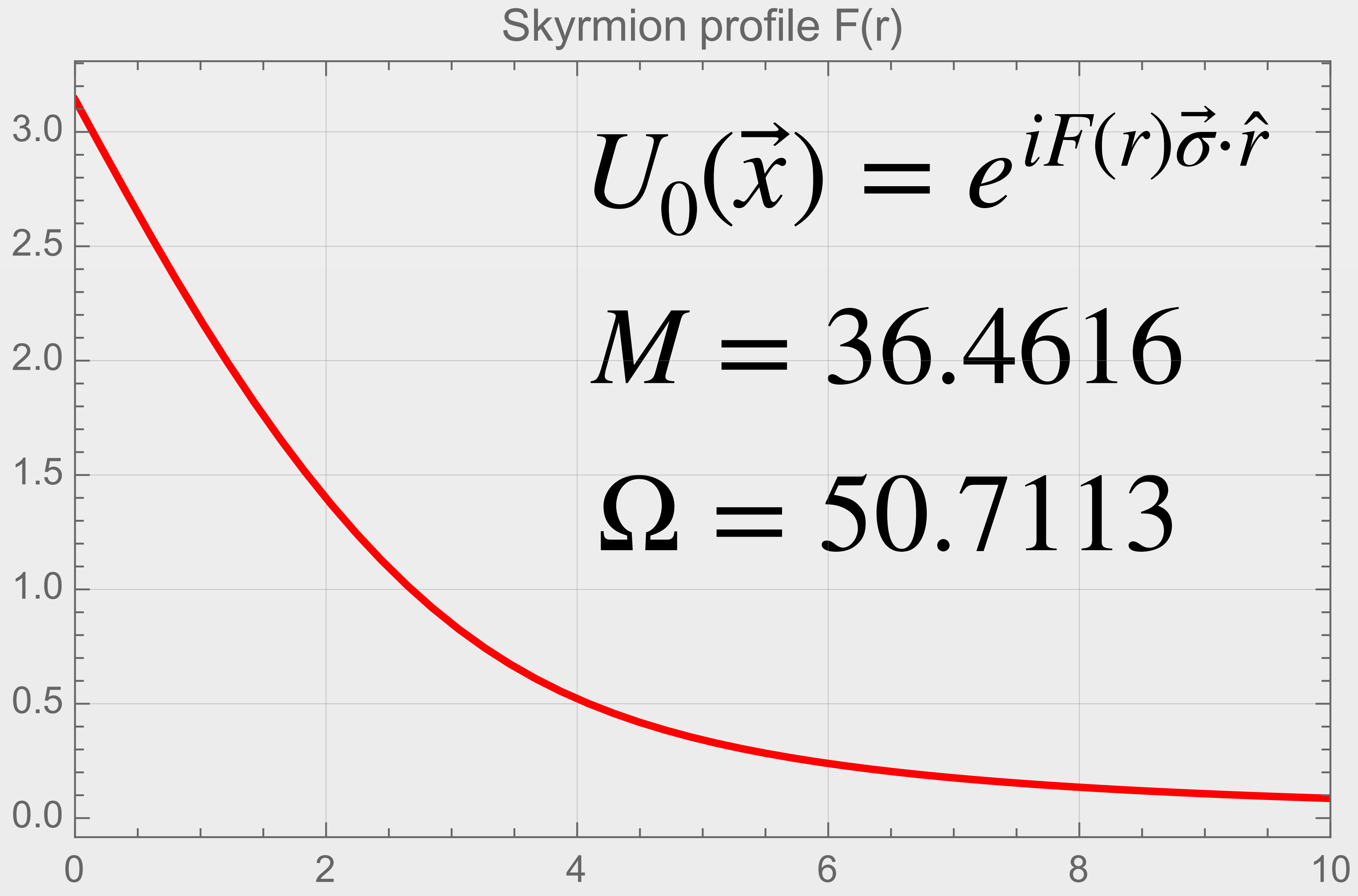

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Nucleon ($I = 1/2$) $\quad m_N = 938\text{MeV}$

Δ Baryon ($I = 3/2$) $\quad m_\Delta = 1232\text{MeV}$

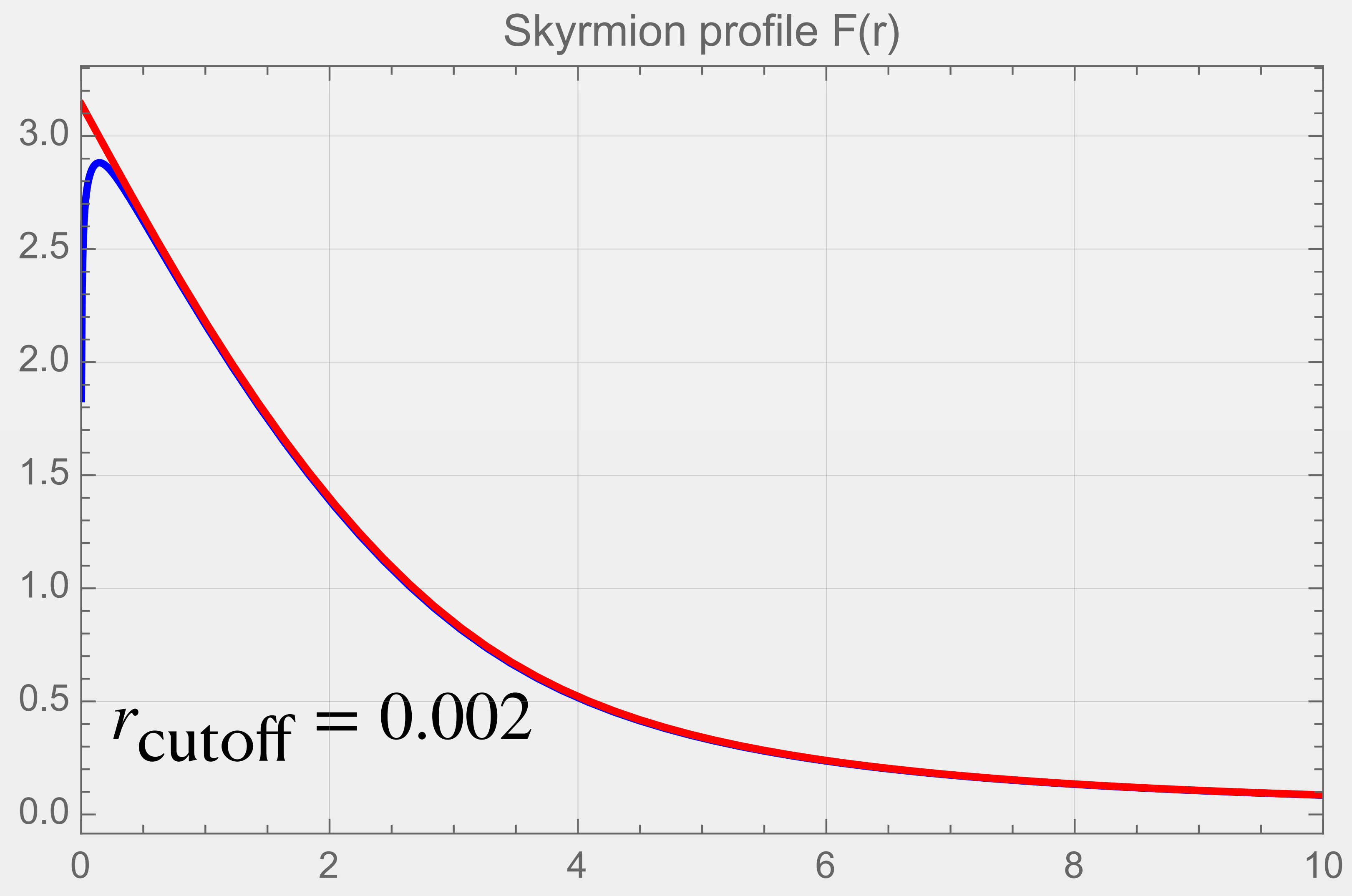
$\longrightarrow \quad f_\pi = 129\text{MeV}, \quad e = 5.44$

Numerics for Fractional Skyrmions

[HTL to appear]

Numerics for Fractional Skyrmions

Bottom Baryon Qqq ($B = 2/3$)



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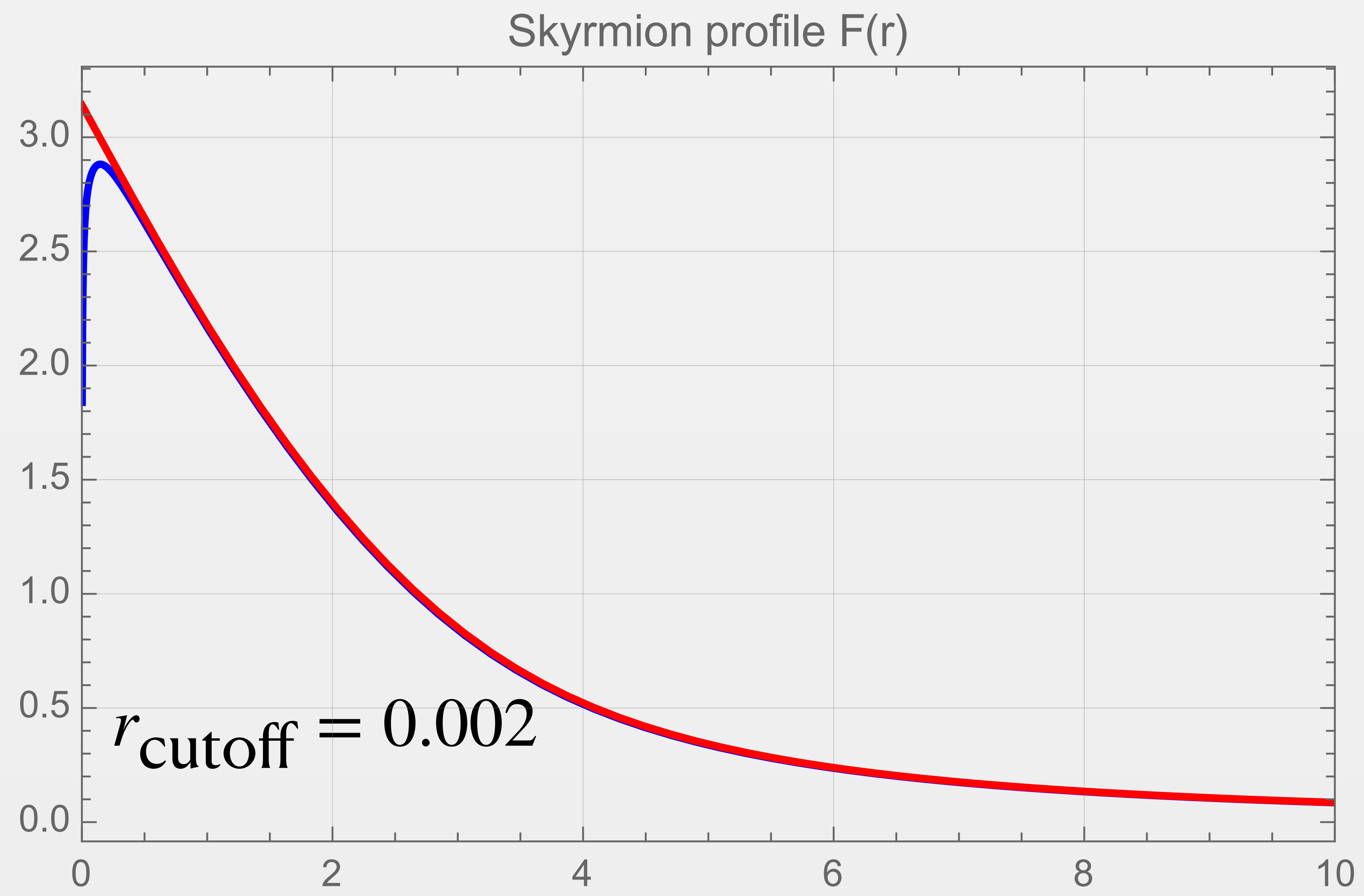

$F(r)$ instantly approaches $B = 1$ skyrmion

[HTL to appear]

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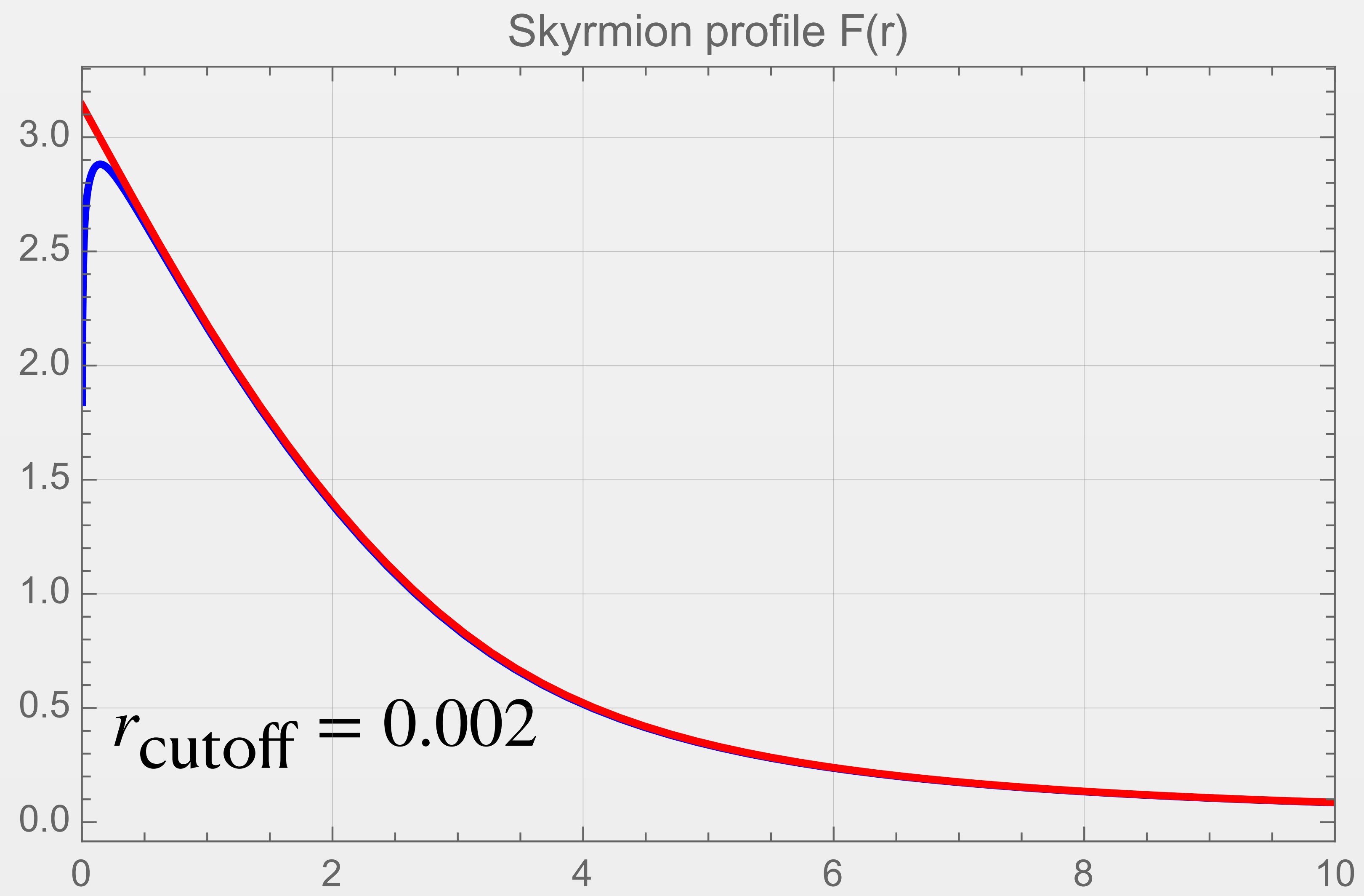

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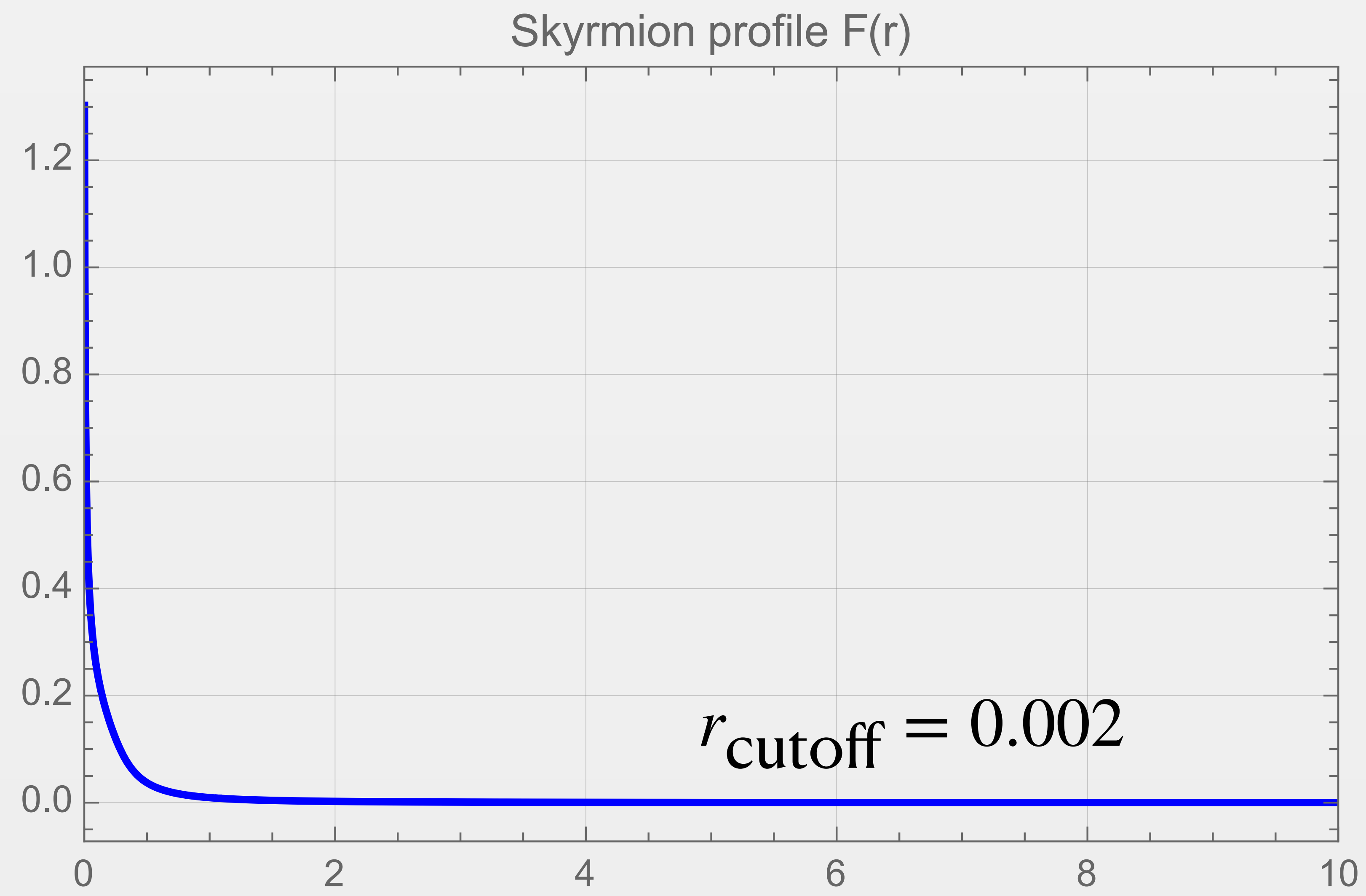

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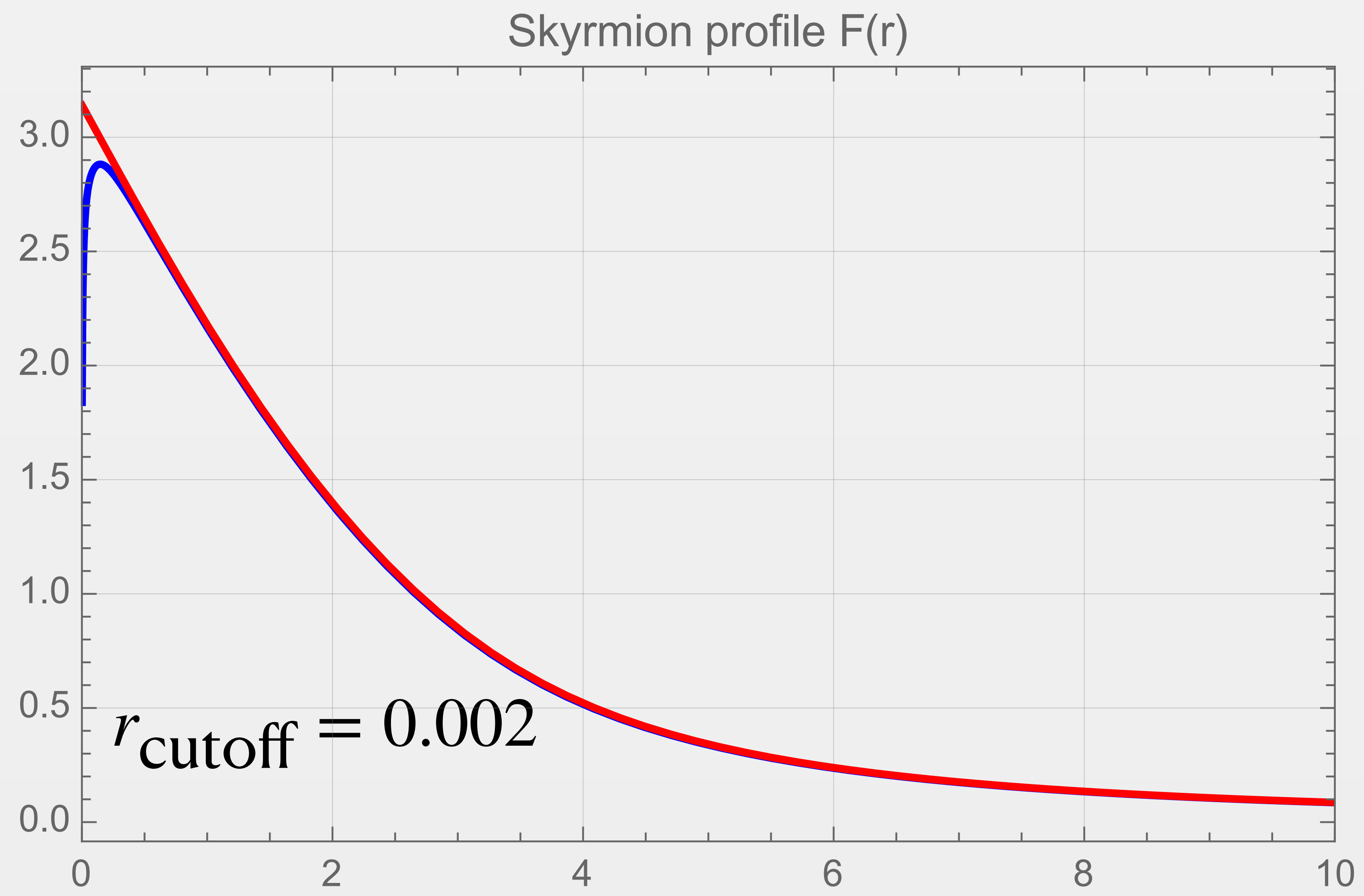

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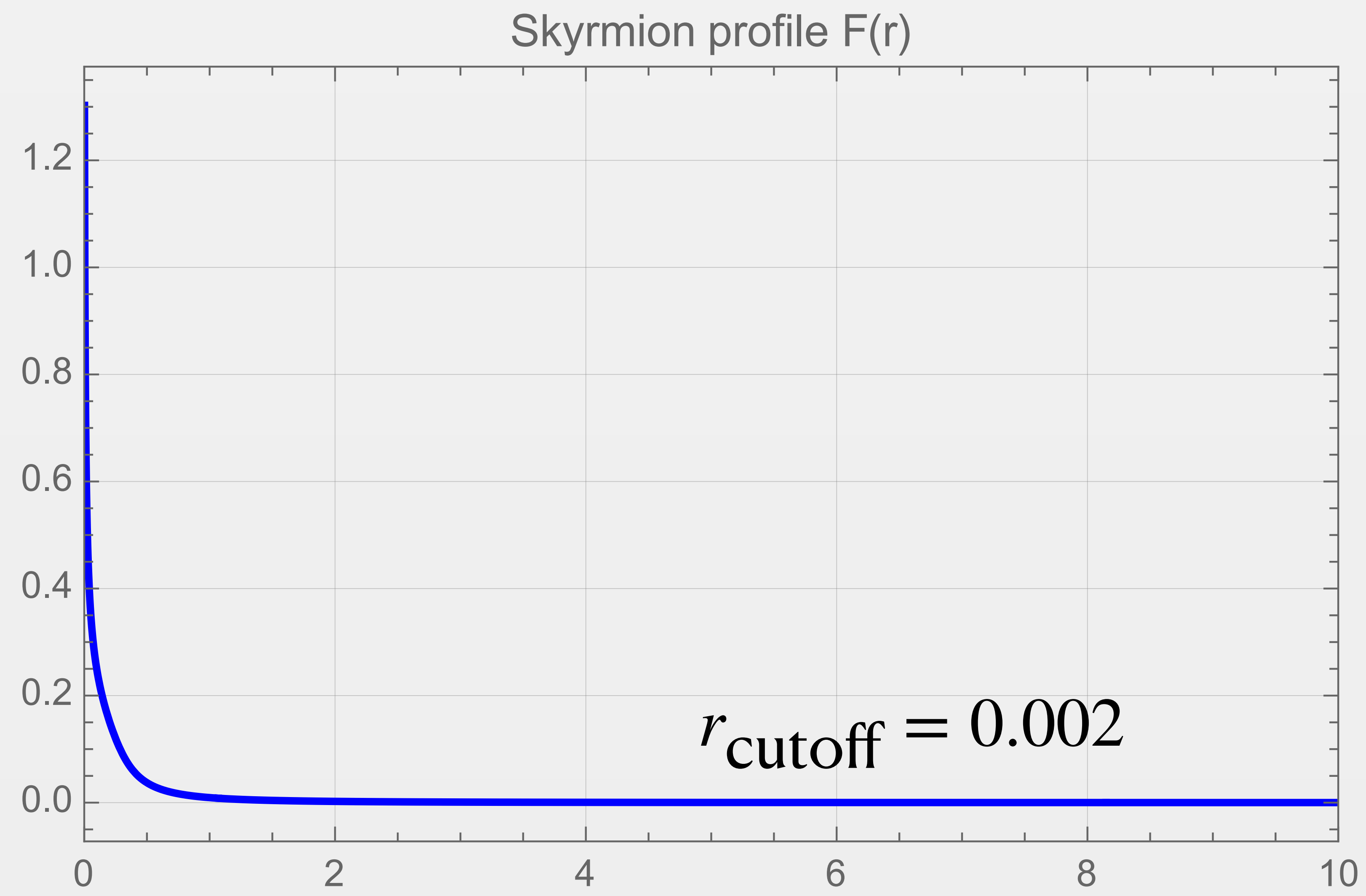

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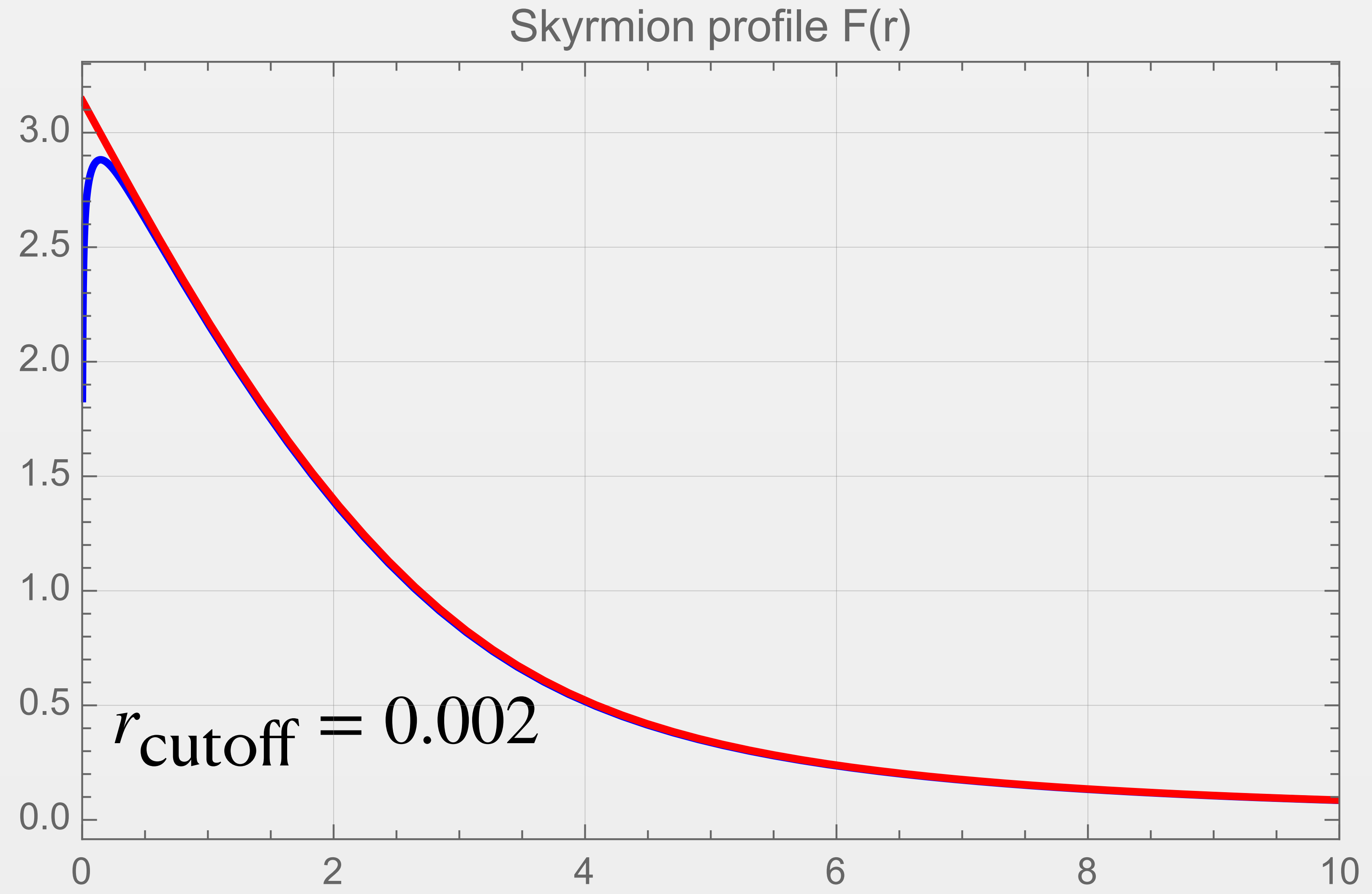

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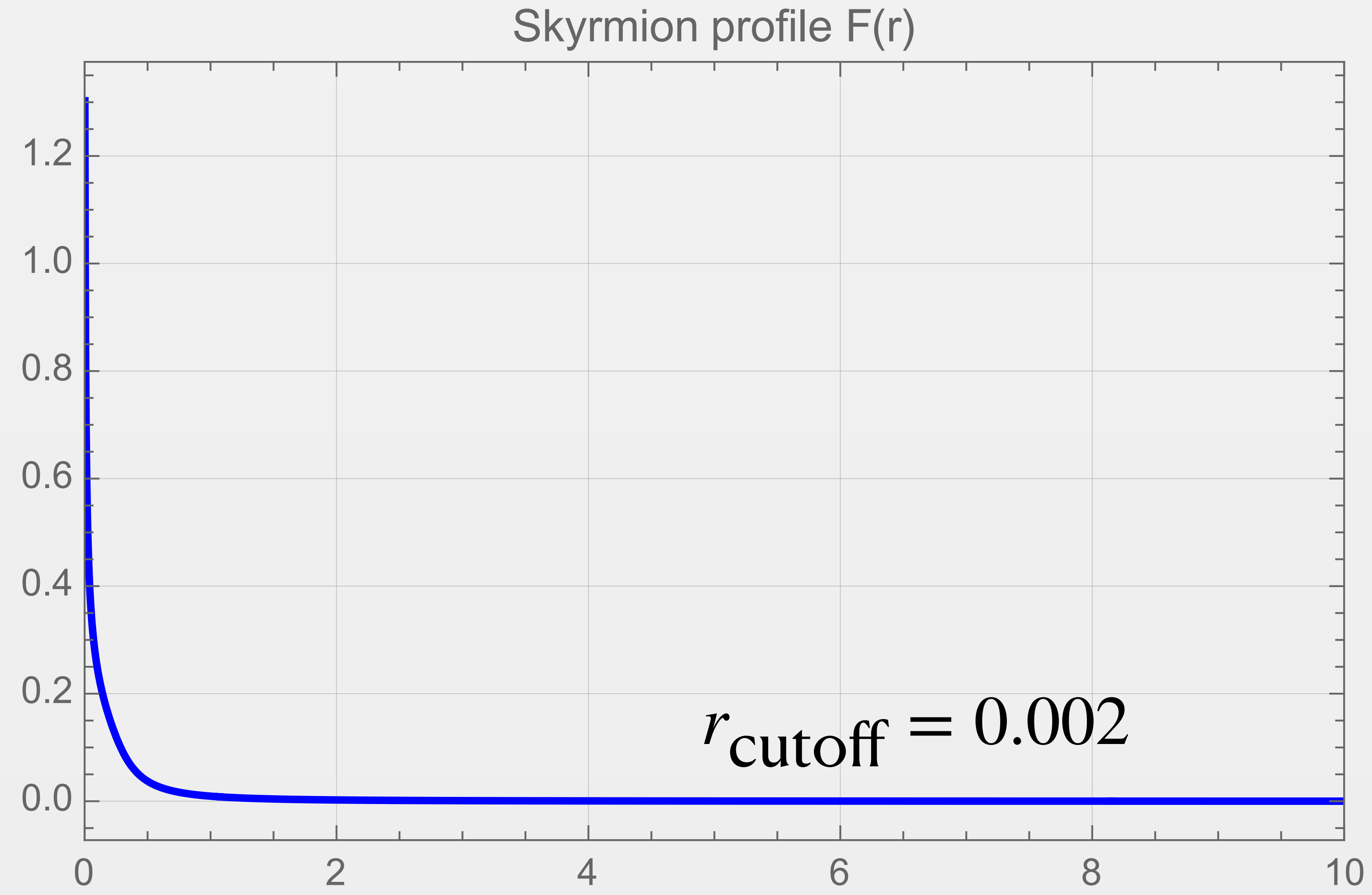

$F(r)$ instantly decays to 0

Bottom baryon appears as a bound state of
a heavy-light meson and a $B = 1$ skyrmion!

Resembles the conventional bound state model of heavy-light hadrons in
[Callan-Klebanov '85, Jenkins-Manohar-Wise '92]

[HTL to appear]

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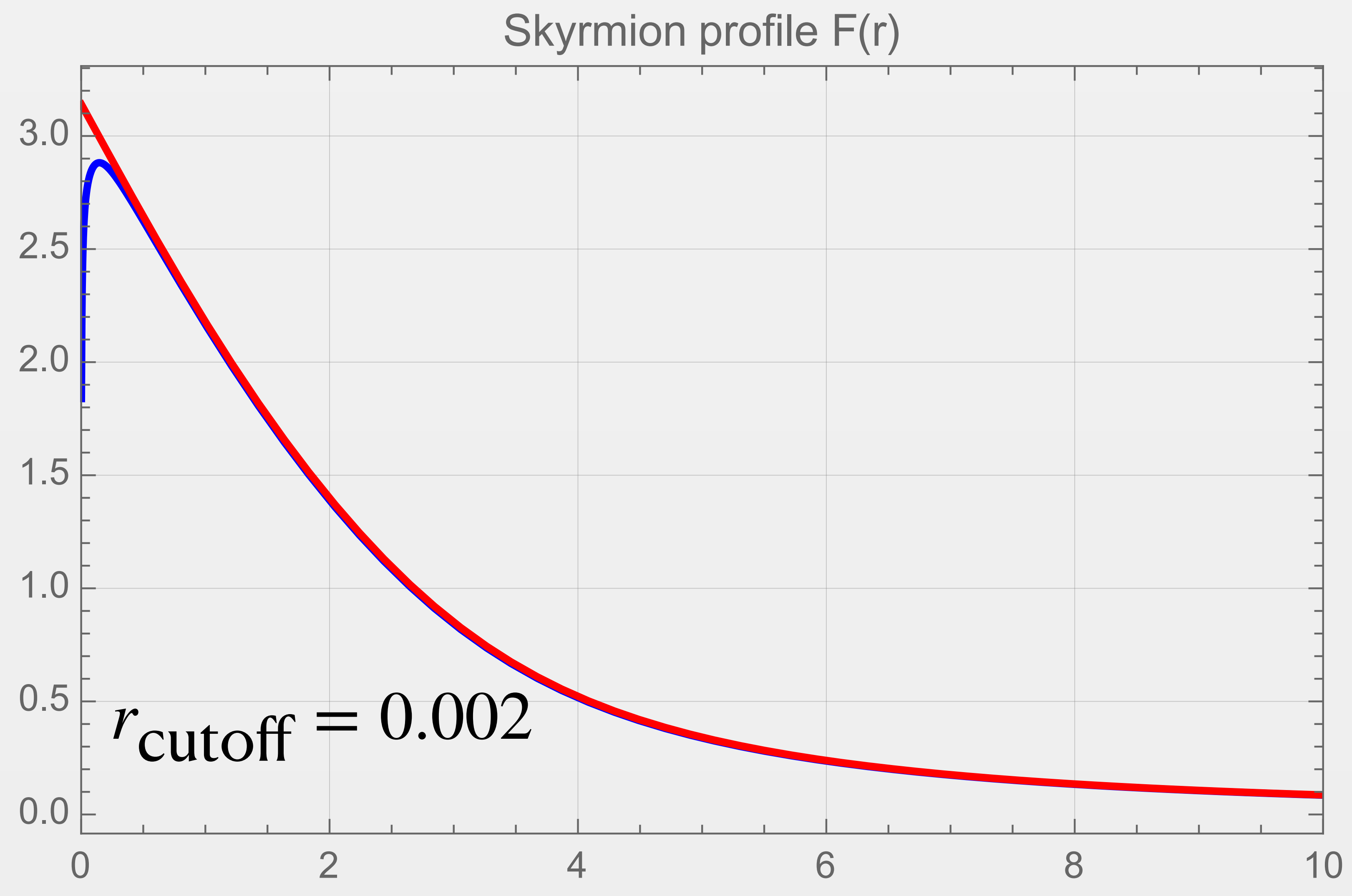

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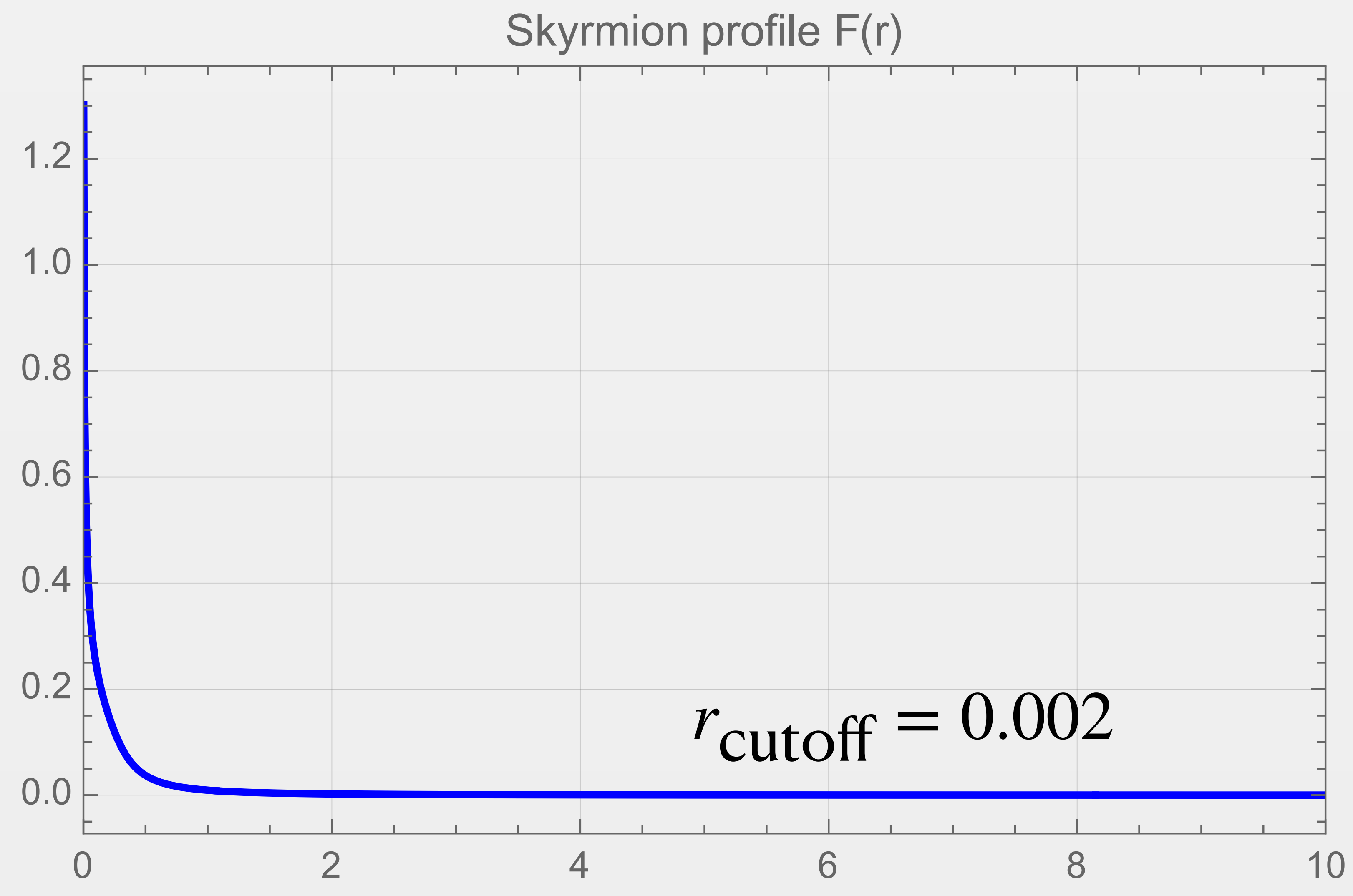

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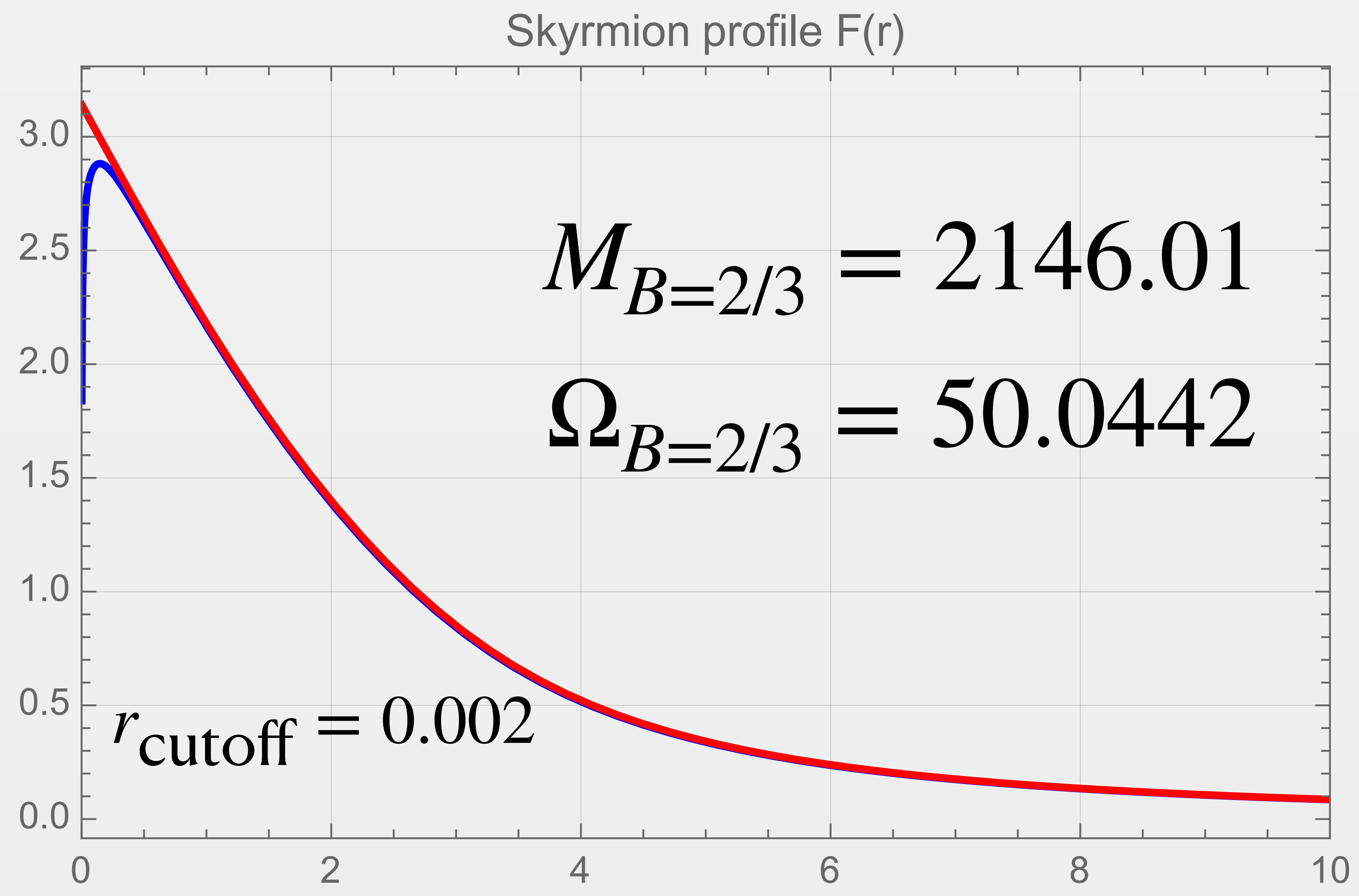

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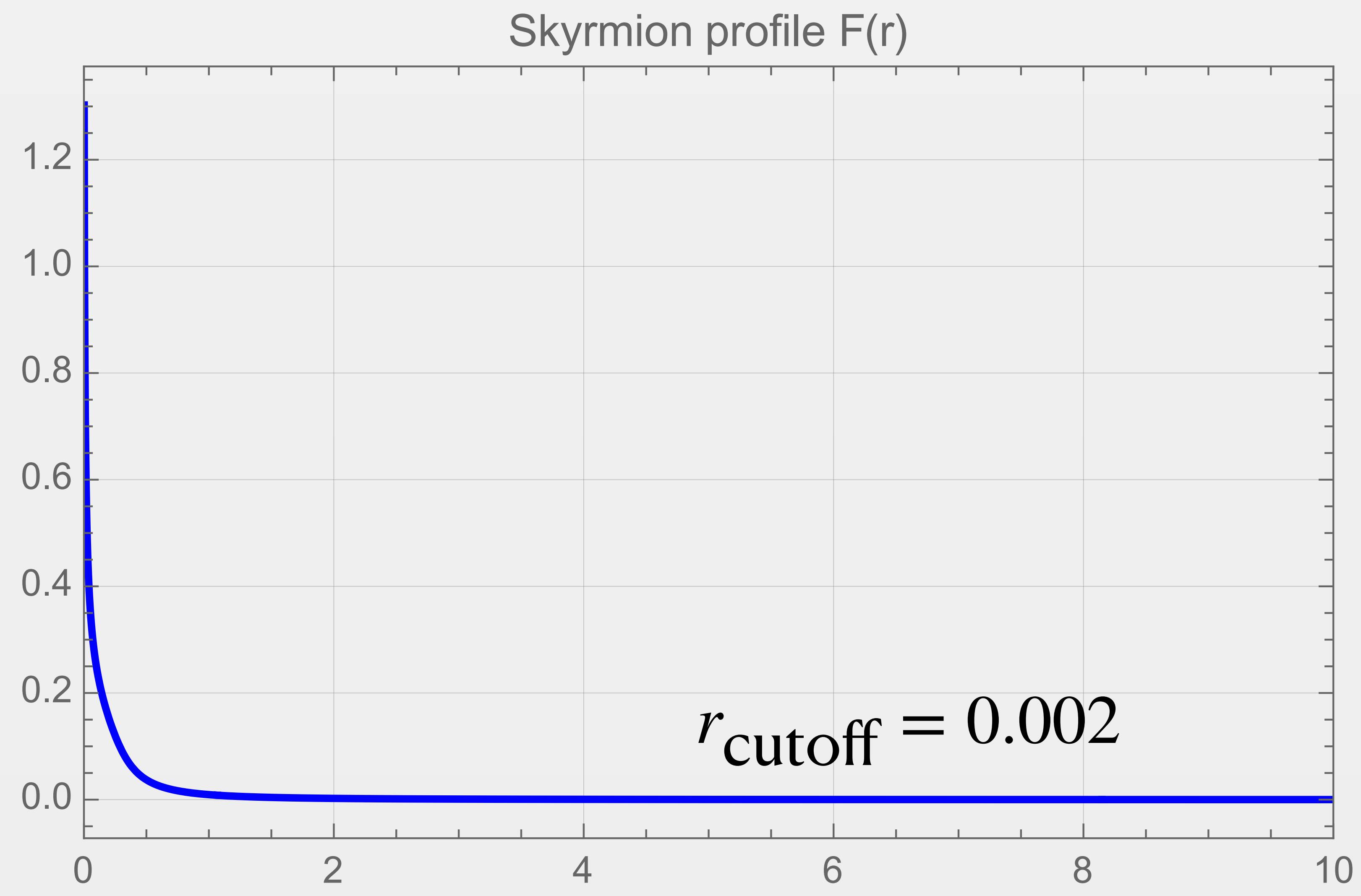

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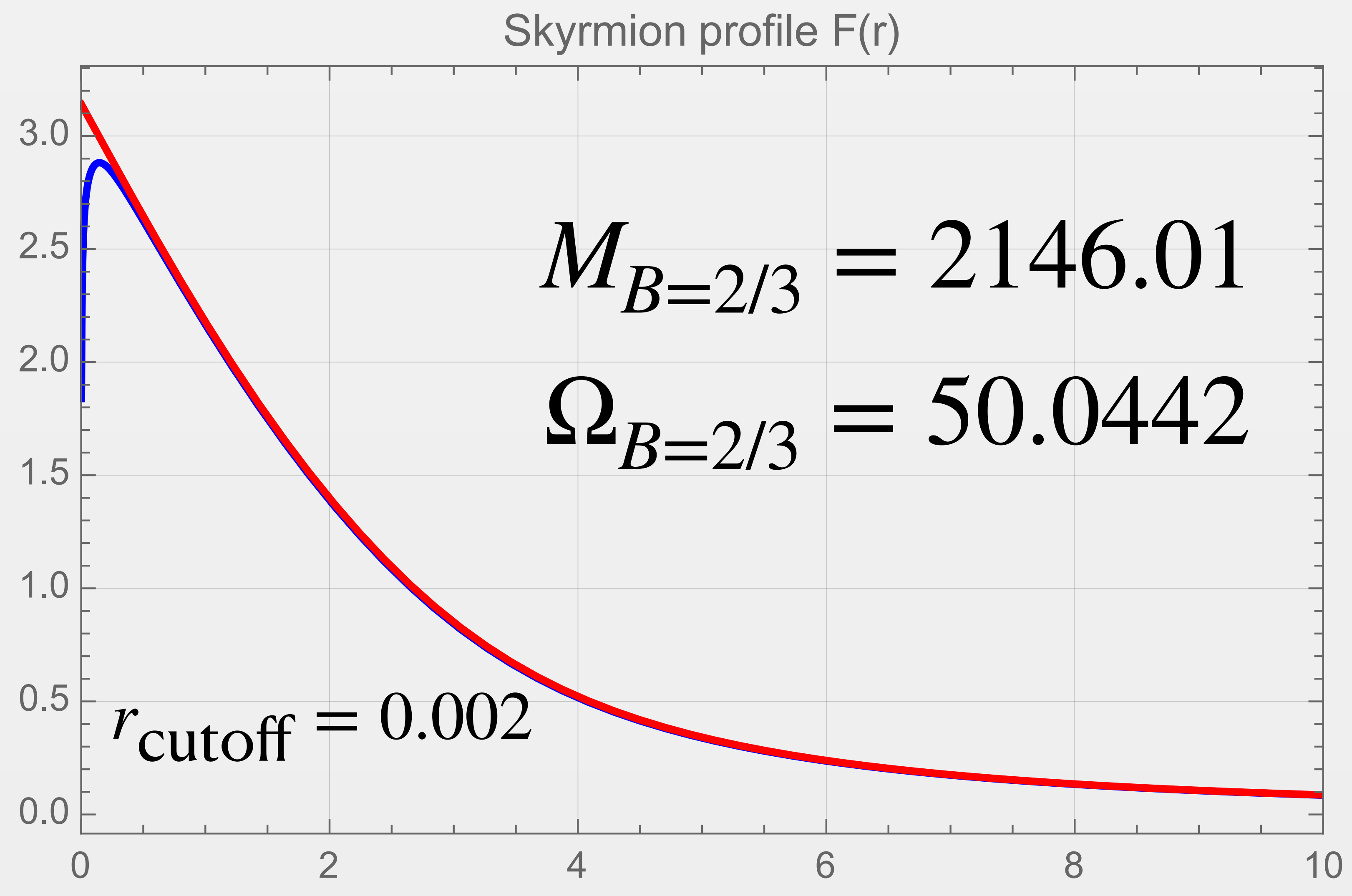

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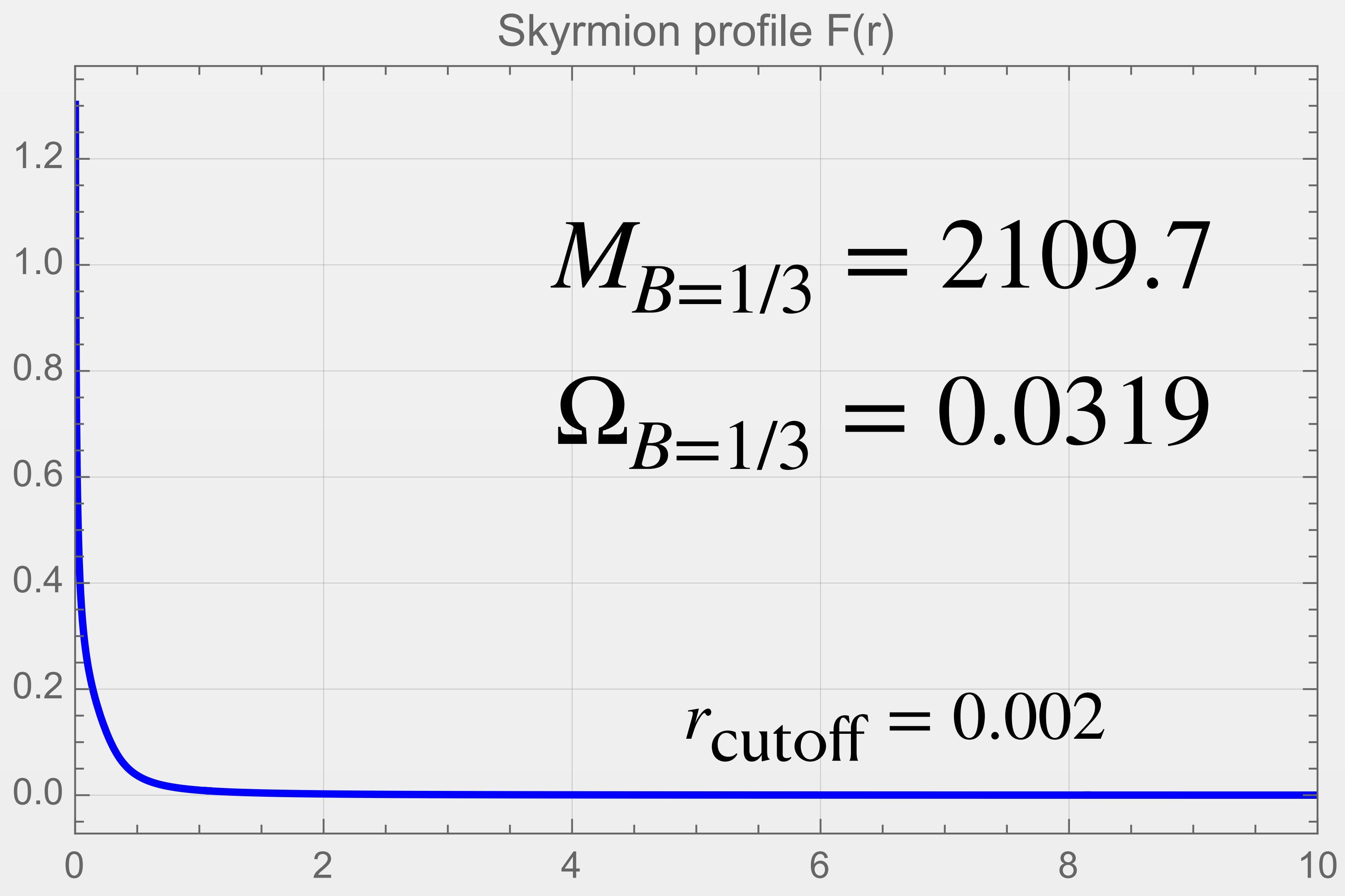

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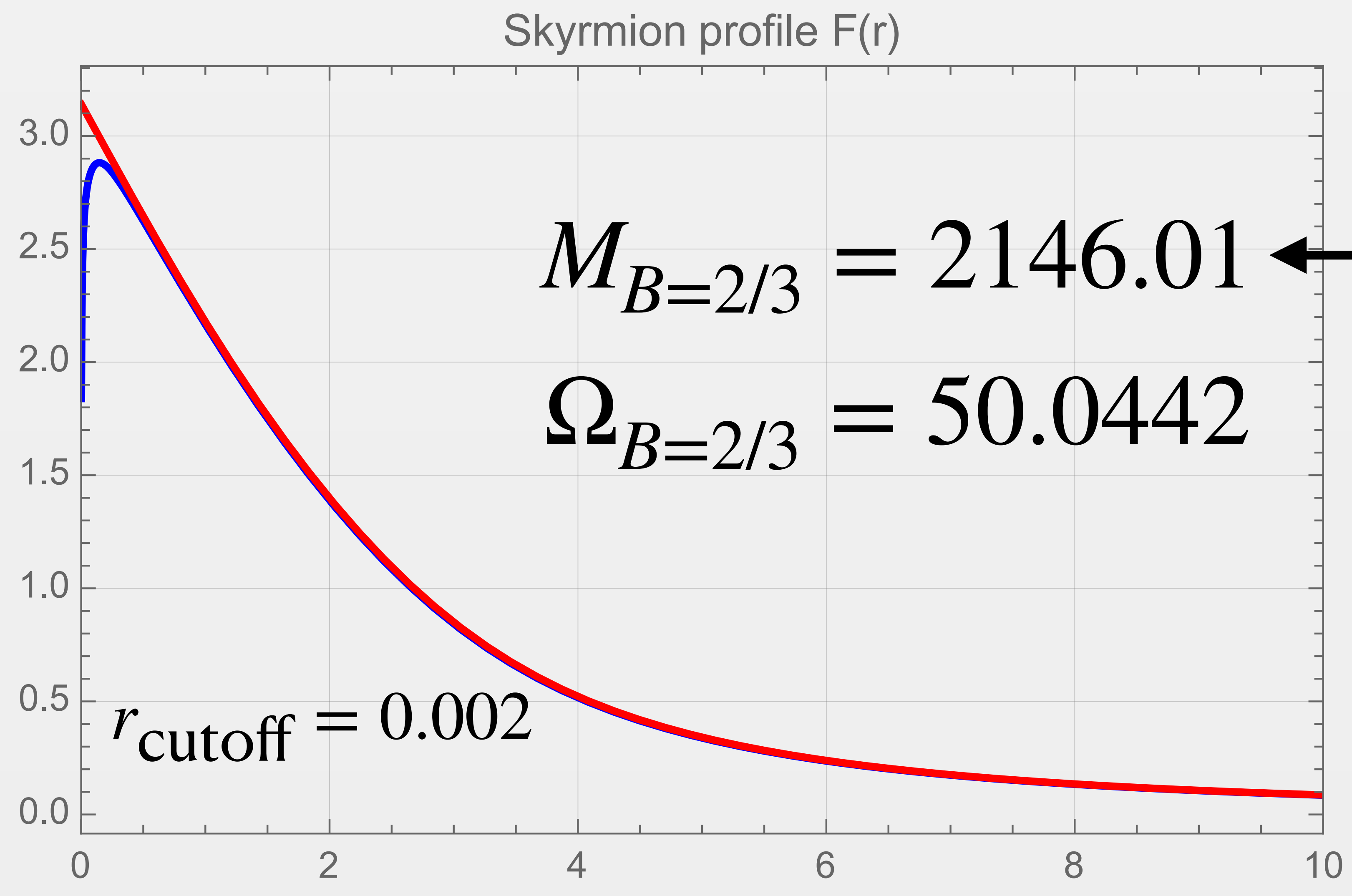

$M_{B=2/3} = 2146.01$

$\Omega_{B=2/3} = 50.0442$

As cutoff approaches 0

diverges with finite difference

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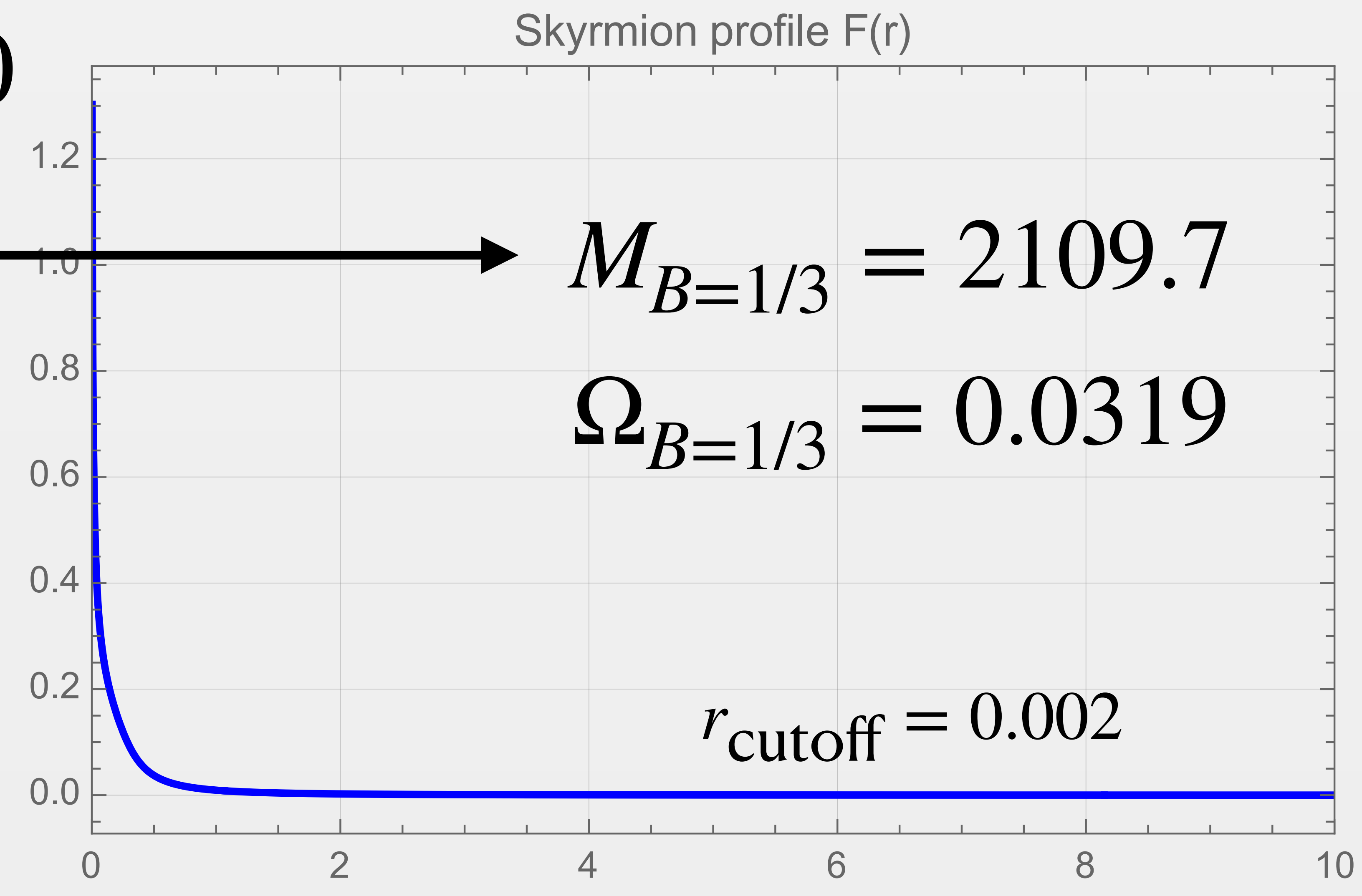

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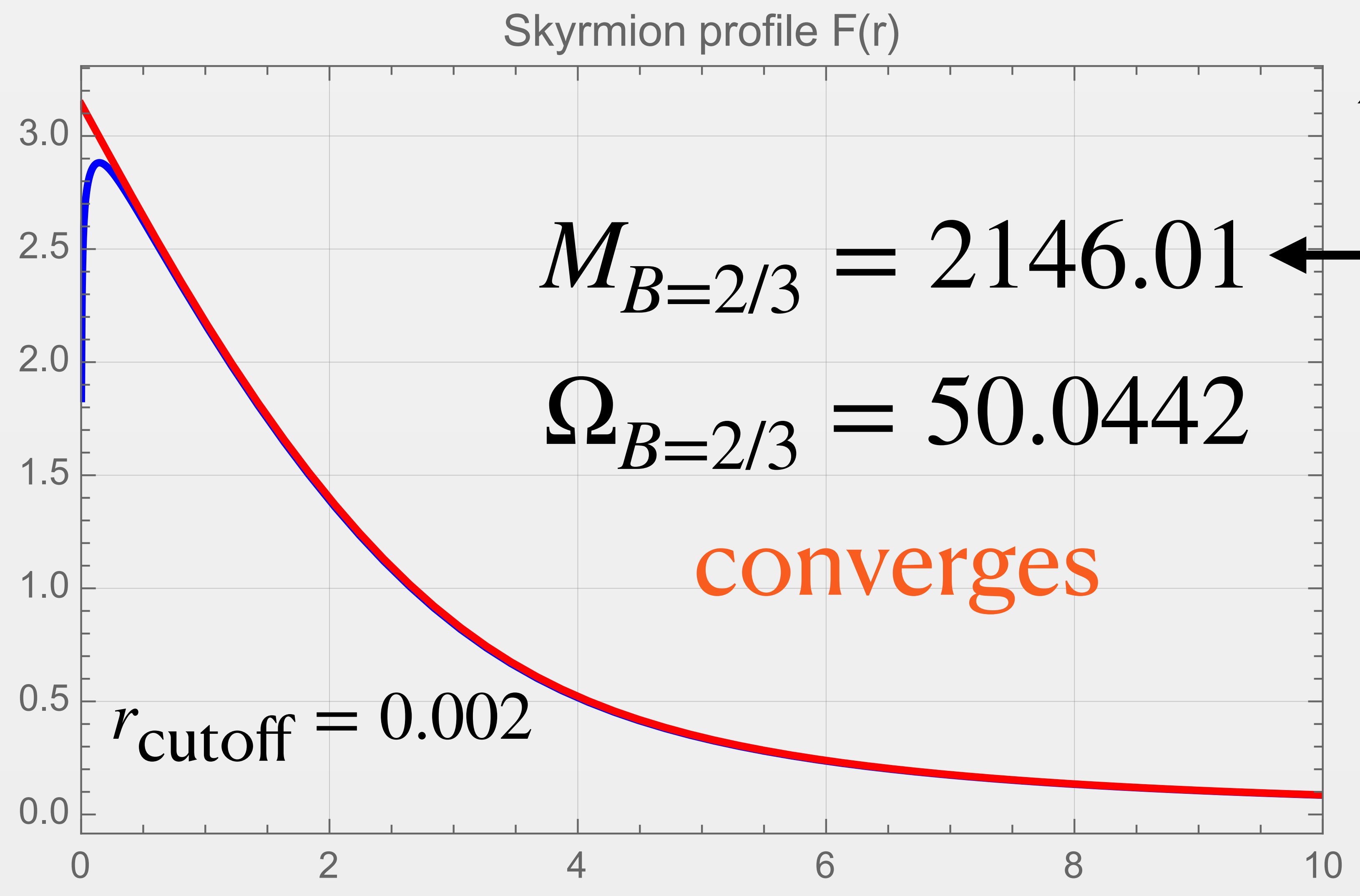

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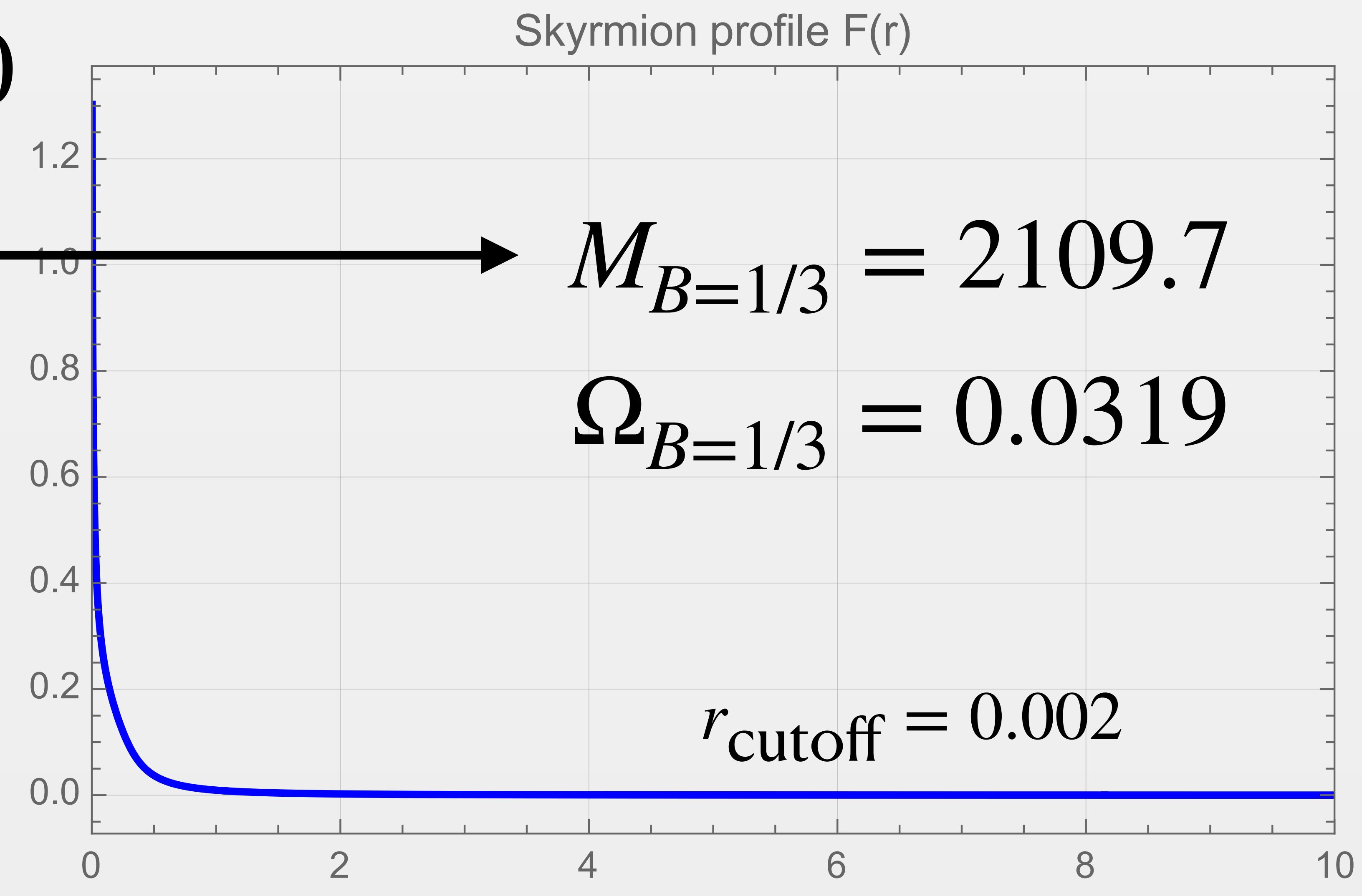

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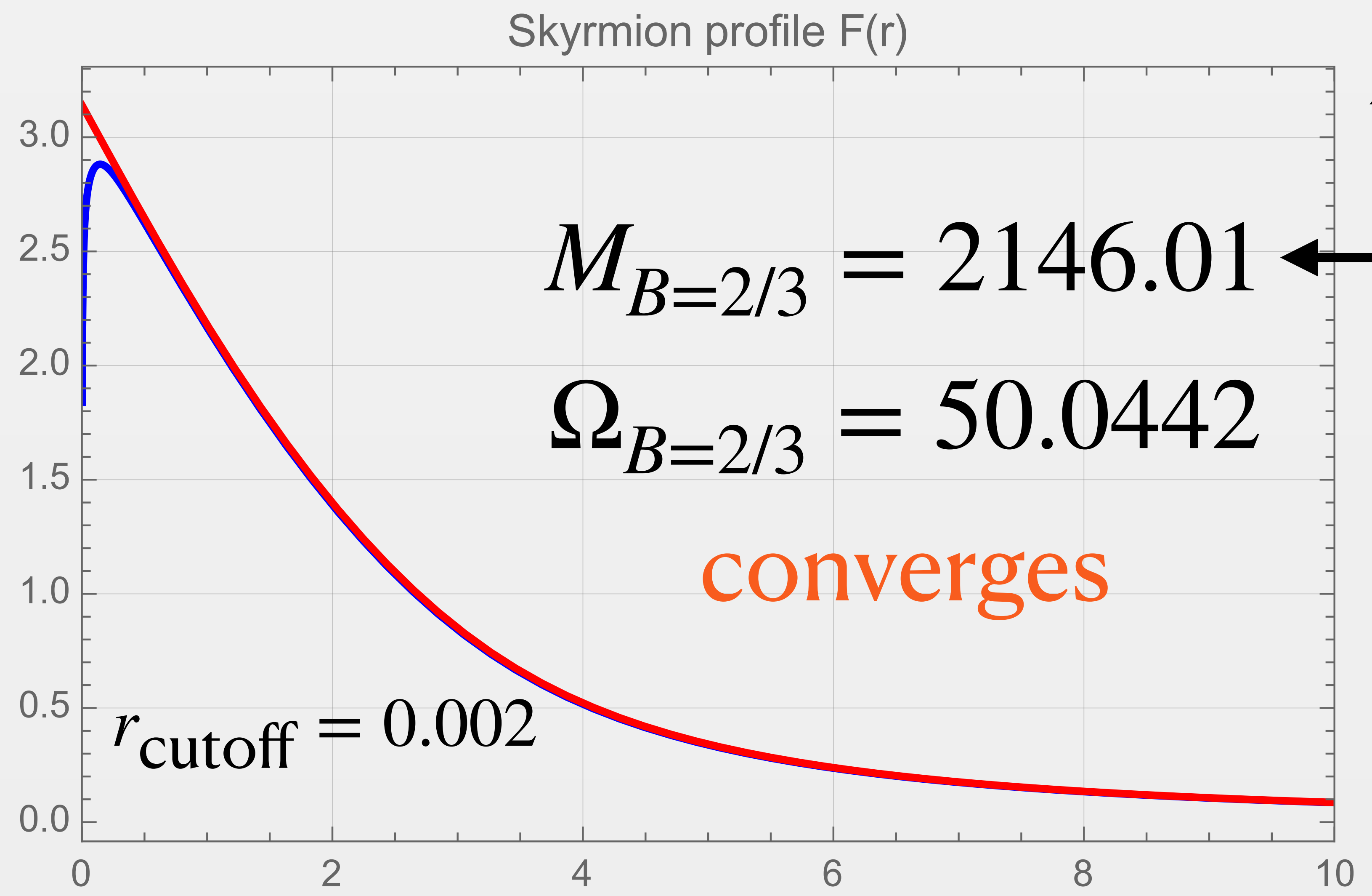

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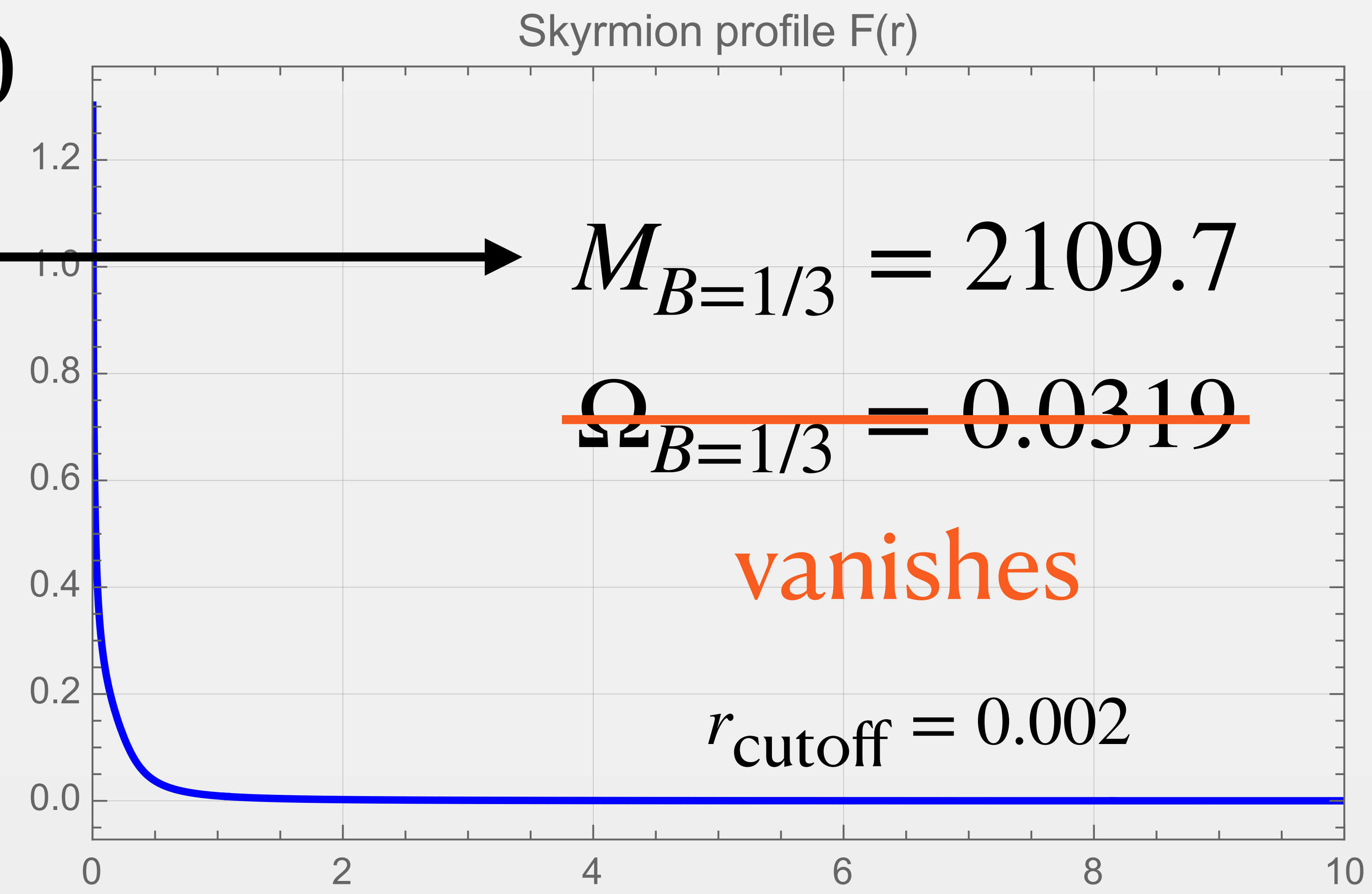

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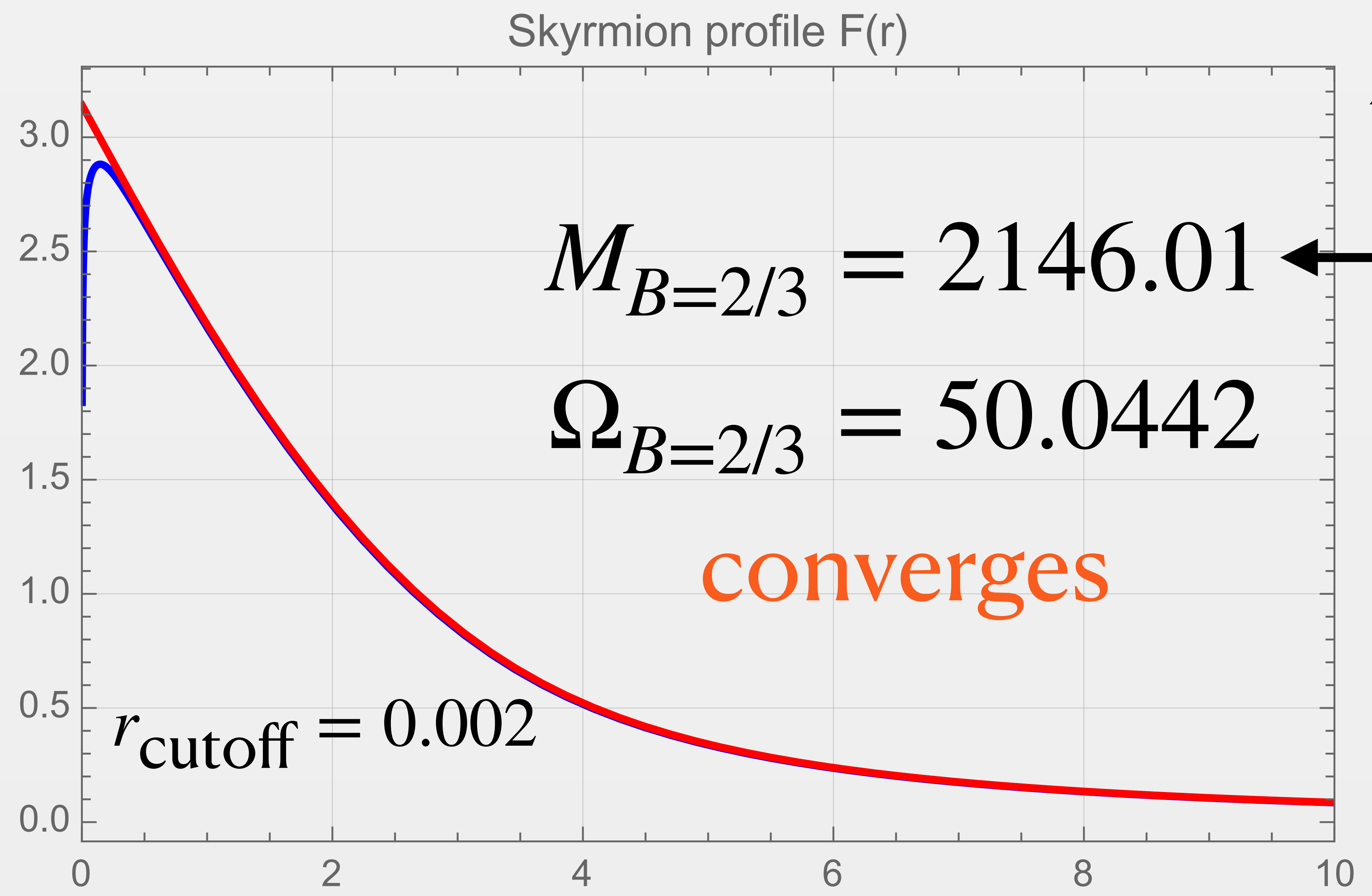

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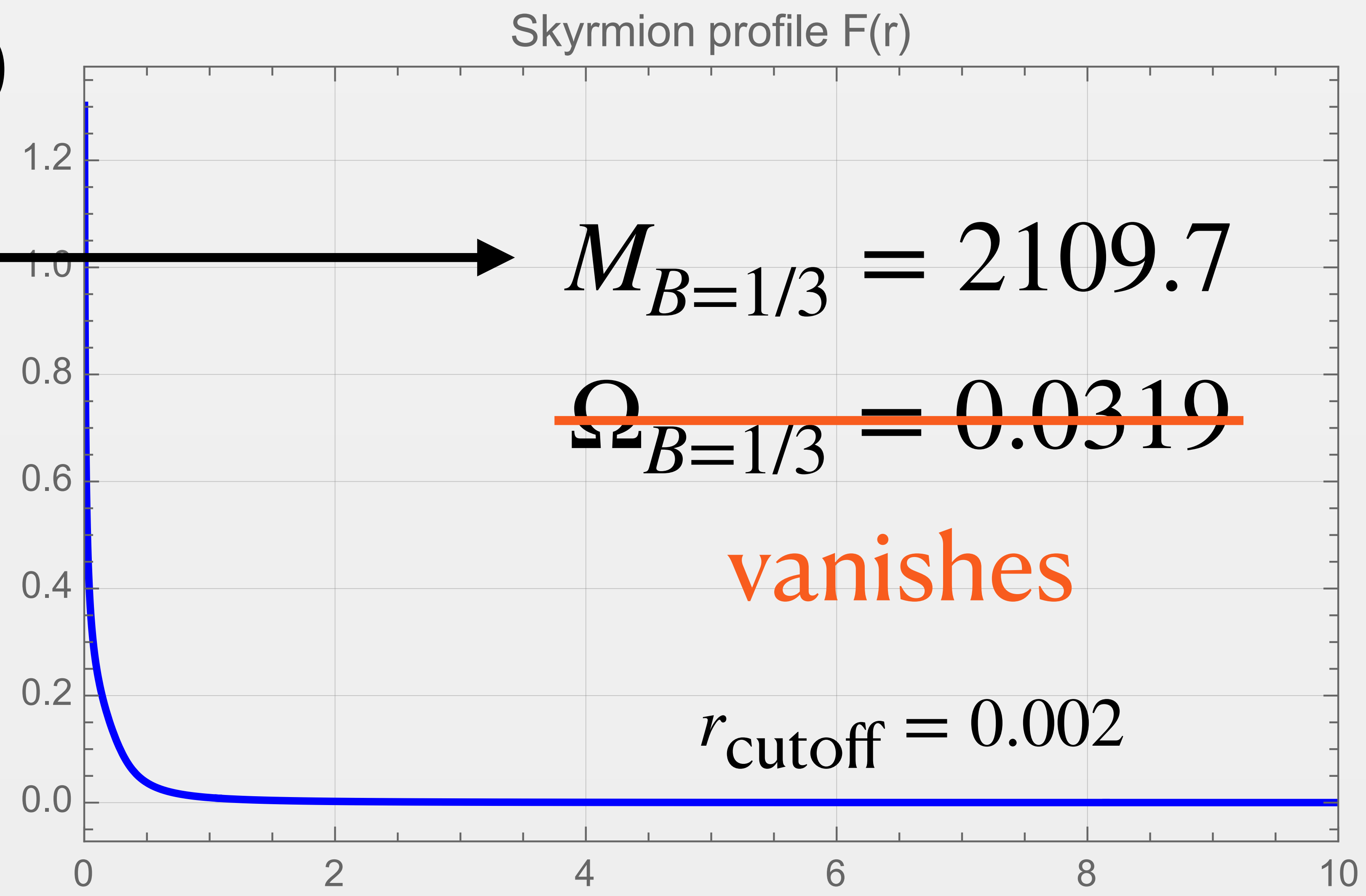

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$$m_\Sigma^{I=1} - m_\Lambda^{I=0} = 198\text{MeV}$$

$$m_{\Sigma_b}^{\exp} - m_{\Lambda_b}^{\exp} = 191\text{MeV}$$

surprisingly close!! 🙂

[HTL to appear]

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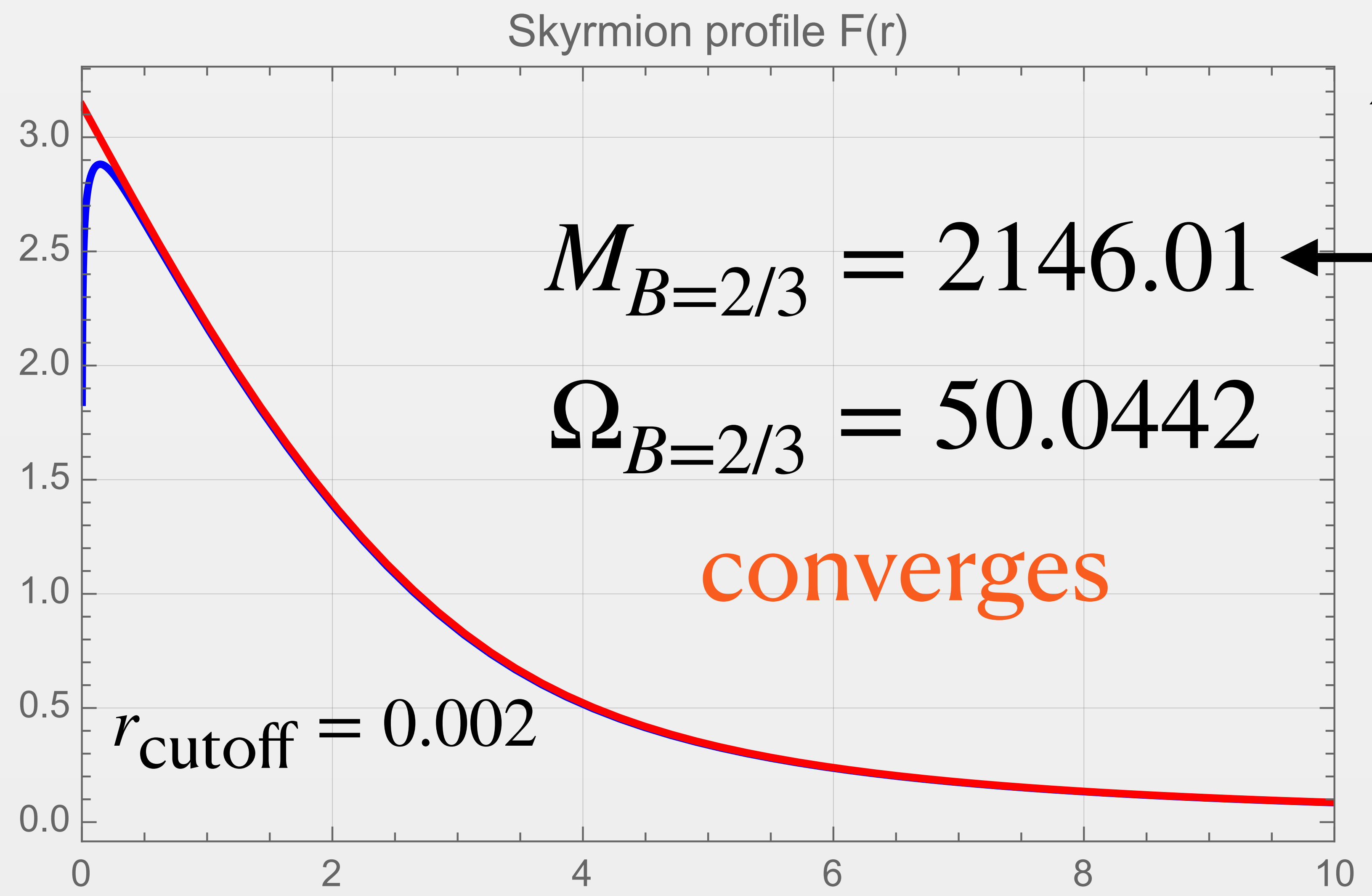

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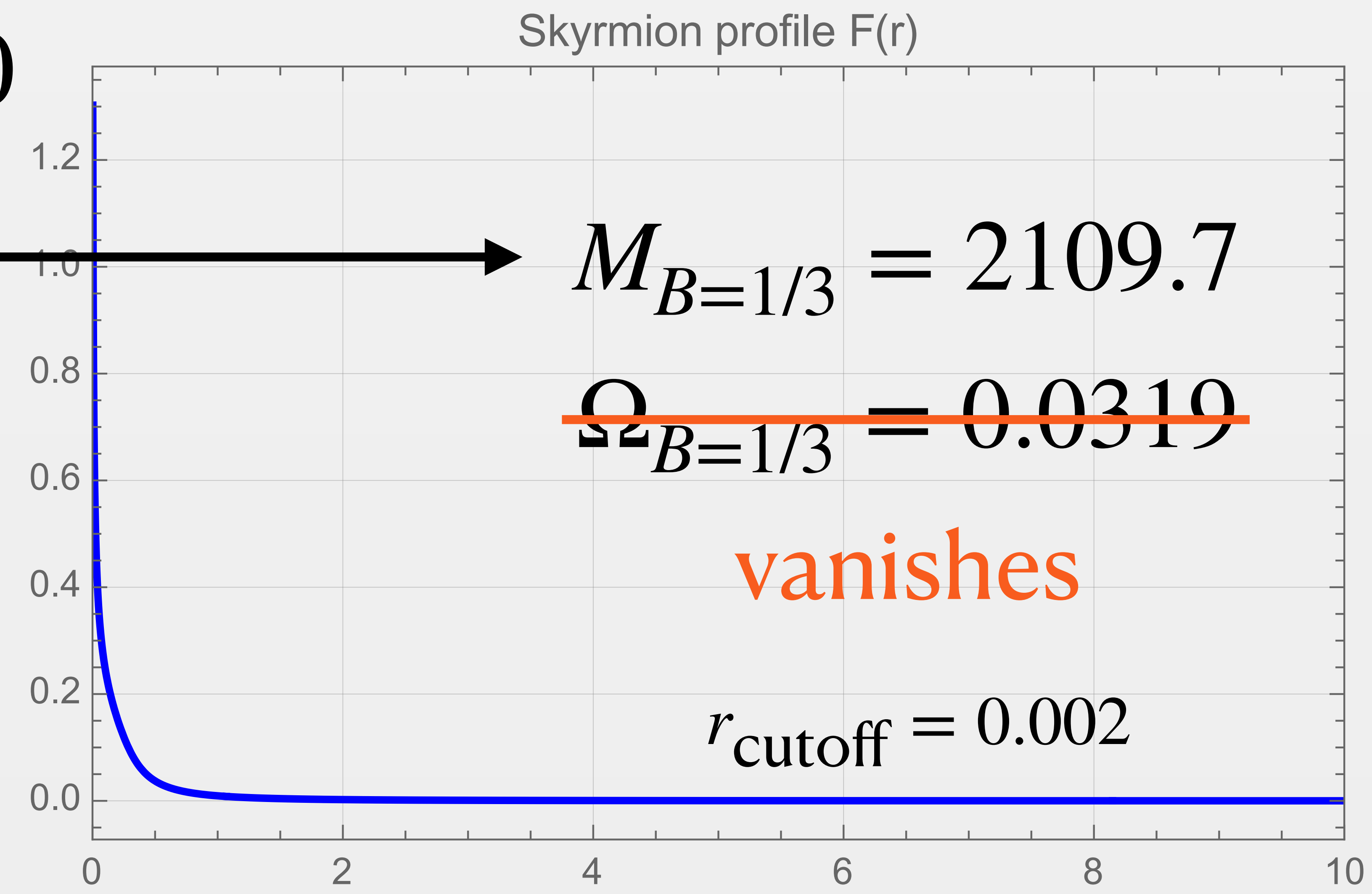

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bound state model [Guralnik-Luke-Manohar '92]

[HTL to appear]

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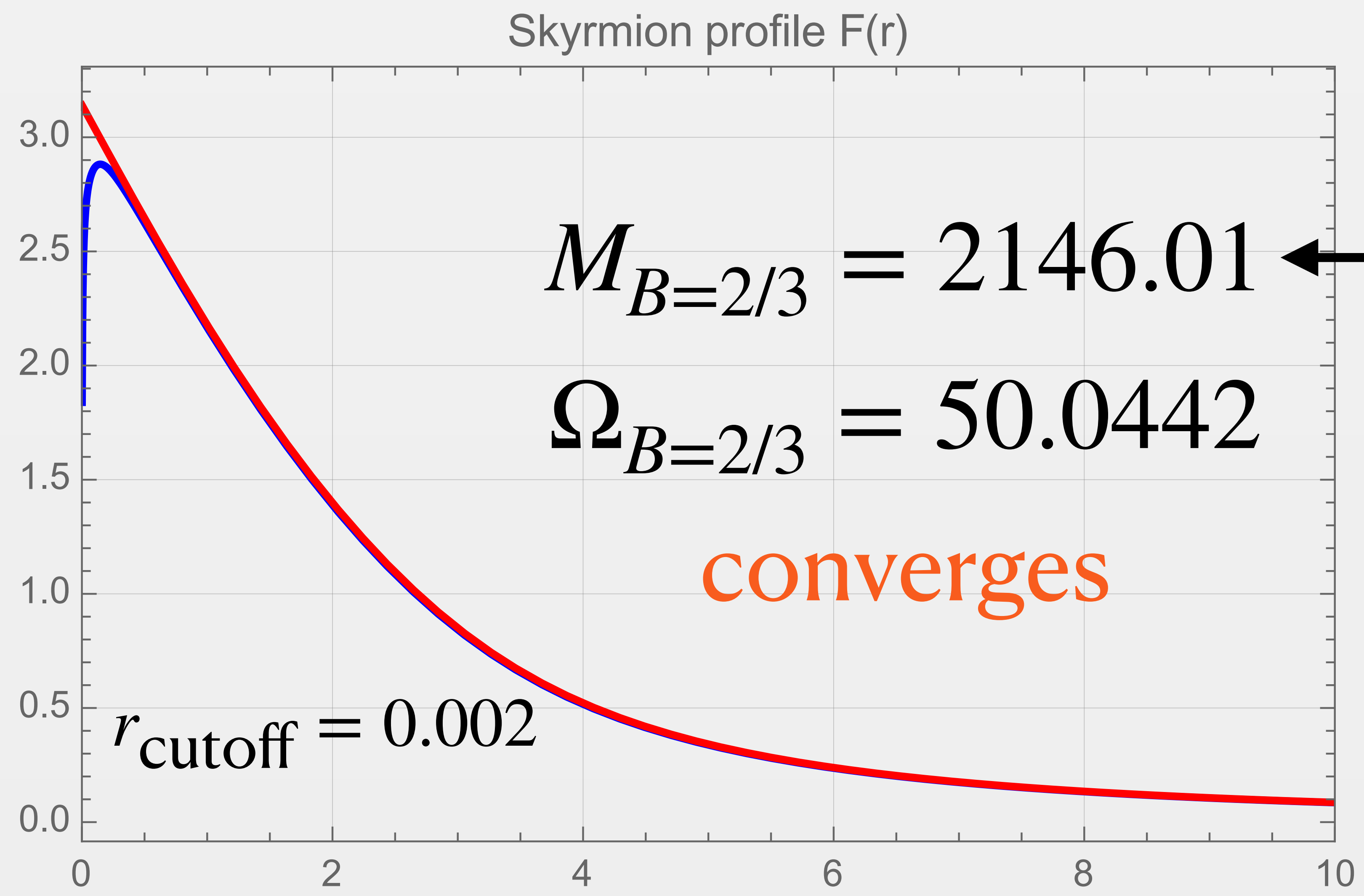

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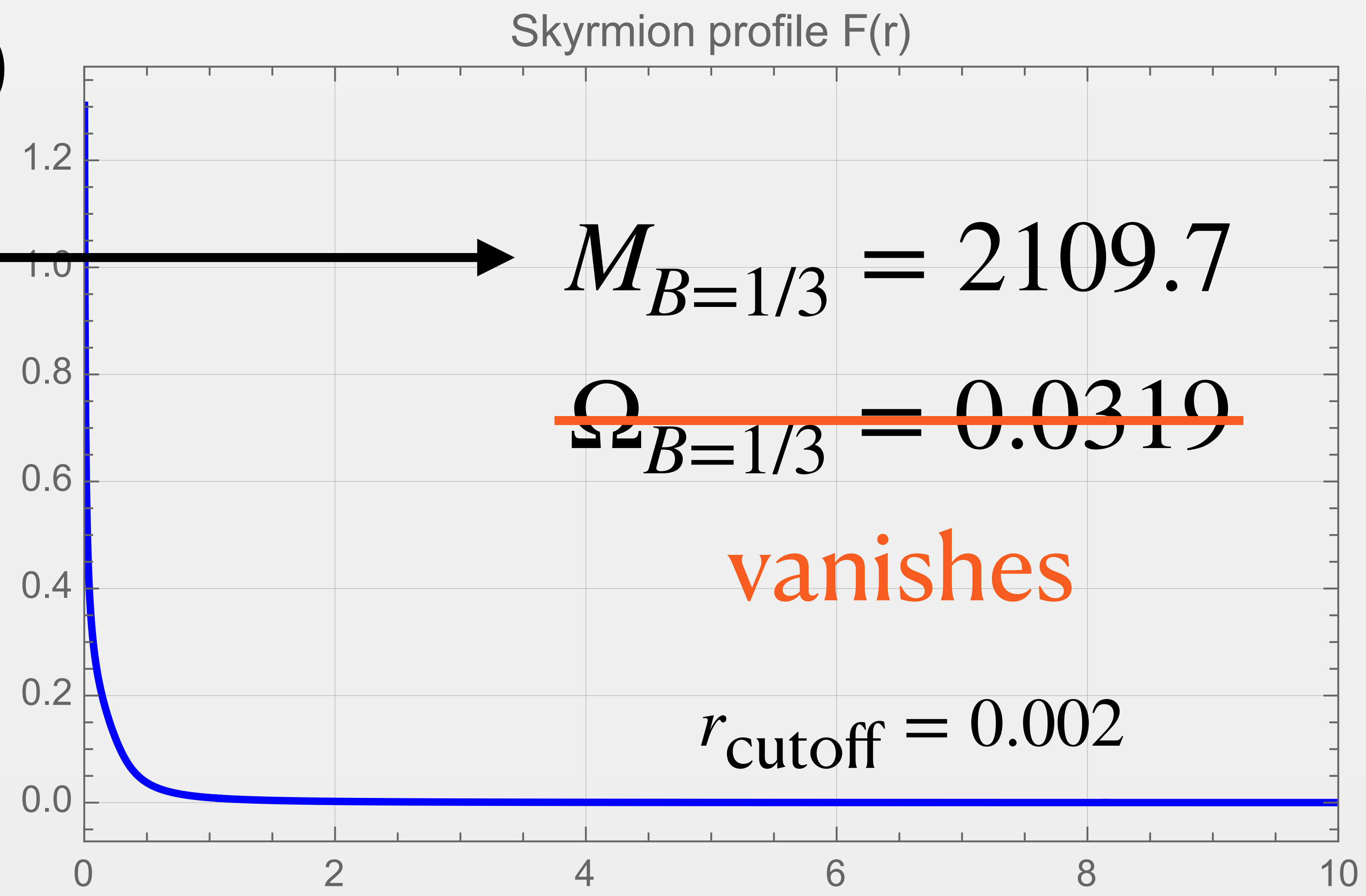

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$$f_\pi (M_{B=2/3} - M_{B=1/3})/e = 861\text{MeV}$$

$$m_{\Lambda_b}^{\text{exp}} - m_B^{\text{exp}} = 340\text{MeV}$$

isospin contribution diverges 🙁

bound state model [Guralnik-Luke-Manohar ‘92]

[HTL to appear]

Heavy Quark Effective Field Theory

See [Manohar-Wise 2000] for review

Heavy Quark Effective Field Theory

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The conventional bound state model of heavy-light hadrons [Callan-Klebanov ’85, Jenkins-Manohar-Wise ’92] was constructed from HQET + chiral Lagrangian.

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New idea for QCD! Many more to explore!

Backup Slides

$N_f = 2$

For $SU(2)$ non-linear σ-model, the Wess-Zumino term vanishes.

What replaces it is a $\mathbb{Z}_2$ θ-term associated with $\pi_4(SU(2)) = \mathbb{Z}_2$.

The non-trivial element in $\pi_4(SU(2)) = \mathbb{Z}_2$ can be constructed by taking an $SU(2)$ skyrmion and rotate it by 2π as it evolves in the time direction.

Another way to think about this $\mathbb{Z}_2$ θ-term is to embed the $SU(2)$ field inside $SU(3)$ and extend it to a 5d bulk. The corresponding Wess-Zumino term gives the θ-term.

The coefficient of the θ-term is $N\pi$ mod 2π

- Matches the Witten anomaly
- Reproduces the statistics of the baryons

In the presence of boundary, the θ-term is no longer $\mathbb{Z}_2$-valued and its coefficient is $N\pi$, which is meaningful not only modulo 2π