

*Symmetries and Anomalies relevant to Symmetric
Mass Generation*

许岑珂

Cenke Xu

Symmetries and Anomalies relevant to Symmetric Mass Generation

Outline:

1. Definition of SMG, 't Hooft anomaly
2. Example of 0+1d SMG, Parity anomaly
3. Examples of 1+1d SMG, relation to Lieb-Shultz-Mattis
4. Example of 2+1d SMG, Parity anomaly
5. Symmetry and anomaly of SMG in 3+1d

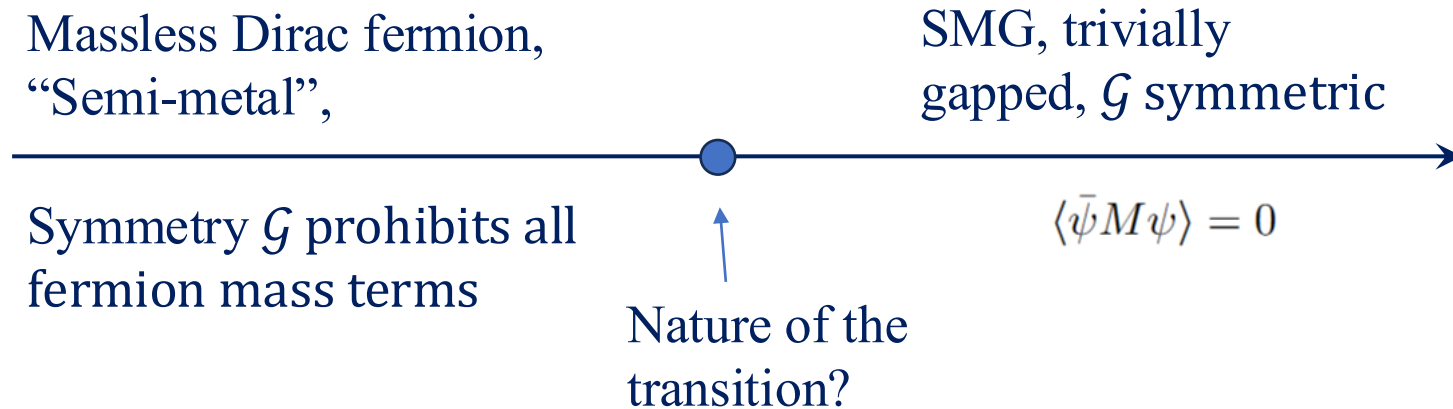
Collaborators: Kevin Slagle, Yi-Zhuang You, Yuan-Yao He, Zi-Hong Liu, Zi-Yang Meng, Fakher Assaad, Anna Hasenfratz, Dan Sehayek, Ho Tat Lam.....

Symmetries and Anomalies relevant to Symmetric Mass Generation

Trivially gapped = symmetric, gapped and nondegenerate (unique GS/vacuum);

Symmetric mass generation: the system has a trivially gapped phase; but there is also a point or phase with (weakly interacting) massless Dirac fermions, and **there is a key symmetry $\mathcal{G}$ under which all fermion-bilinear mass operators transform nontrivially.**

$$\mathcal{G} (\bar{\psi} M \psi) \mathcal{G}^{-1} = -\bar{\psi} M \psi$$



The gapless phase often has some large emergent and also anomalous symmetry G_{IR} in the IR. In SMG phase, this G_{IR} symmetry must be broken. “S” in SMG refers to “ $\mathcal{G}$ ”, not G_{IR} .

Symmetries and Anomalies relevant to Symmetric Mass Generation

Trivially gapped = symmetric, gapped and nondegenerate (unique GS/vacuum);

Only possible when the system has no 't Hooft anomaly.

For SMG we need to ensure:

- (1) there is a symmetry $\mathcal{G}$ under which all fermion mass operators transform nontrivially;
- (2) There is no self 't Hooft anomaly for $\mathcal{G}$, and there is **no mixed anomaly between $\mathcal{G}$ and other exact symmetries.**

Remark 1: Anomaly cancellation is only the **necessary** but **not sufficient** condition for SMG.

Anomaly cancellation means that SMG in principle can happen, but it does not answer whether a specific model (action, Hamiltonian) realizes SMG or not.

Remark 2: With 't Hooft anomaly, the ground state degeneracy can come from either spontaneous symmetry breaking, or topological order.

Symmetries and Anomalies relevant to Symmetric Mass Generation

Remark 3: three different “things” called anomaly:

- 1, Gauge anomaly for a dynamical gauge field: the worst kind of anomaly, the theory should not exist (no way to realize this theory).
- 2, ABJ-anomaly for a symmetry G means that the symmetry is explicitly broken (by instanton).
- 3, **‘t Hooft anomaly** is much more “benign”, in fact it is a powerful tool: the symmetry is still valid, and the theory is still consistent.

But ‘t Hooft anomaly of G means three things:

- 1, Constraint on how the symmetry is realized on the lattice;
- 2, Obstruction to gauging (most systematic diagnosis of ‘t Hooft anomaly): once we gauge the symmetry G , there will be gauge anomaly; or gauging G_1 will necessarily break G_2 , means there is a mixed ‘t Hooft anomaly between G_1 and G_2 .
- 3, the spectrum above the ground state cannot be trivially gapped.

Symmetries and Anomalies relevant to Symmetric Mass Generation

Self 't Hooft anomaly of the key symmetry.

dimension	(0+1)	(1+1)	(2+1)	(3+1)
$\mathcal{G}$	$\mathcal{T}, \mathcal{T}^2 = 1$	Spin- Z_4 ,	$\mathcal{T}, \mathcal{T}^2 = -1$	Spin- Z_4
Anomaly class	$\mathbb{Z}_4$	$\mathbb{Z}_4$,	$\mathbb{Z}_8$	$\mathbb{Z}_8$

Remark 4: in all cases, the smallest element is a single complex Dirac fermion. The classification will be different if the smallest element is taken to be Majorana, or Weyl fermion.

Remark 5: For complex fermions, Spin- Z_4 is equivalent to chiral- Z_2 if we combine Spin- Z_4 with $U(1)_V$.

Symmetries and Anomalies relevant to Symmetric Mass Generation

Example of SMG in 2+1d

Key symmetry is a nonunitary $\mathcal{G} = CT$. One can verify that, with CT , all Dirac fermion mass operators change sign:

$$\mathcal{G}(\bar{\psi}M\psi)\mathcal{G}^{-1} = -\bar{\psi}M\psi$$

The self 't Hooft anomaly of CT is cancelled for multiple of 8 Dirac fermions, or 16 Majorana fermions ($\mathbb{Z}_{16}$ of ${}^3\text{He-B}$).

Relation to interacting topological insulator/superconductor.

Many entries reduces from $\mathbb{Z}$ to $\mathbb{Z}_4, \mathbb{Z}_8 \dots$ Under interaction. Meaning, for 8 copies of AIII topological insulator (or 16 copies of ${}^3\text{He-B}$), its 2+1d boundary **cannot be trivially gapped without interaction**, but **can be trivially gapped with interaction**, precisely SMG.

Class	$d = 1$	$d = 2$	$d = 3$	$d = 4$
A	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AIII	$\mathbb{Z} \rightarrow \mathbb{Z}_4$	0	$\mathbb{Z} \rightarrow \mathbb{Z}_8$	0
AI	0 F&K, 2009	0	0	$\mathbb{Z}$
BDI	$\mathbb{Z} \rightarrow \mathbb{Z}_8, \mathbb{Z} \rightarrow \mathbb{Z}_4$	0	0	0
D	$\mathbb{Z}_2$	$\mathbb{Z}$	0 ${}^3\text{He-B}$	0
DIII	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z} \rightarrow \mathbb{Z}_{16}$	0
AII	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$

Symmetries and Anomalies relevant to Symmetric Mass Generation

Self 't Hooft anomaly of the key symmetry.

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$\mathcal{G}$	$\mathcal{T}, \mathcal{T}^2 = 1$	Spin- Z_4 ,	$\mathcal{T}, \mathcal{T}^2 = -1$	Spin- Z_4
Anomaly class	$\mathbb{Z}_4$	$\mathbb{Z}_4$,	$\mathbb{Z}_8$	$\mathbb{Z}_8$

Remark 6: In this talk we will assume that the self-anomaly of $\mathcal{G}$ is cancelled, we will **focus on the potential anomaly between $\mathcal{G}$ and other symmetries G_V** , we also assume there is no self-anomaly of G_V .

Symmetries and Anomalies relevant to Symmetric Mass Generation

Choice of the key symmetry $\mathcal{G}$ (**minimal symmetry** under which **all** fermion-bilinear mass operators transform nontrivially): In 0+1d and 2+1d, most natural choice of $\mathcal{G}$, is CT .

Example: in 0+1d, consider arbitrary numbers of complex fermions.

$$CT : c_a \rightarrow c_a^\dagger, \quad i \rightarrow -i,$$

$$c^\dagger M c \rightarrow c M^* c^\dagger = -c^\dagger M^\dagger c = -c^\dagger M c.$$

For 1+1d and 3+1d, the most natural choice of $\mathcal{G}$ is Spin-Z₄.

$$\psi \rightarrow e^{ik \frac{\pi}{2} \gamma_5} \psi, \quad k = 0, 1, 2, 3.$$

$$\bar{\psi}_a M_{ab} \psi_b, \quad i \bar{\psi}_a \gamma_5 M_{ab} \psi_b$$

Is $\mathcal{G}$ a symmetry at all?

In many lattice system, $\mathcal{G}$ is often realized as an **exact non-onsite** symmetry. For example, chiral-Z₂ is often realized as translation (shown later).

Symmetries and Anomalies relevant to Symmetric Mass Generation

Example of SMG in 0+1d:

In 0+1d, most natural choice of $\mathcal{G}$, is CT .

$$CT : c_a \rightarrow c_a^\dagger, \quad i \rightarrow -i,$$

$$c^\dagger M c \rightarrow c M^* c^\dagger = -c^\dagger M^\dagger c = -c^\dagger M c.$$

Based on the classification table, with total number of **2 complex fermions**, there is self 't Hooft anomaly of CT , cannot have SMG. Luckily, we can show this exactly.

The only CT invariant Hamiltonian:

$$H = U \left(c_1^\dagger c_1 - \frac{1}{2} \right) \left(c_2^\dagger c_2 - \frac{1}{2} \right)$$

GS is two-fold degenerate,
spontaneously breaks CT :

$$|\text{GS}\rangle = |10\rangle \text{ and } |01\rangle, \text{ for } U > 0$$

Symmetries and Anomalies relevant to Symmetric Mass Generation

Example of SMG in 0+1d:

In 0+1d, most natural choice of $\mathcal{G}$, is CT .

$$CT : c_a \rightarrow c_a^\dagger, \quad i \rightarrow -i,$$

$$c^\dagger M c \rightarrow c M^* c^\dagger = -c^\dagger M^\dagger c = -c^\dagger M c.$$

with total number of **4 complex fermions**, no self 't Hooft anomaly of CT , can have SMG. Again we can show this exactly.

$$H = -U (c_1 c_2 c_3 c_4 + h.c.), \quad \rightarrow \quad |\text{GS}\rangle = \frac{1}{\sqrt{2}} (|0000\rangle + |1111\rangle) \quad \langle \text{GS} | c^\dagger M c | \text{GS} \rangle = 0$$

The GS is trivially gapped and symmetric with this Hamiltonian.

Symmetries and Anomalies relevant to Symmetric Mass Generation

Mixed anomaly between $\mathcal{G}$ and flavor symmetry in 0+1d:

In 0+1d, most natural choice of $\mathcal{G}$, is CT .

$$H = -U (c_1 c_2 c_3 c_4 + h.c.), \quad \rightarrow \quad |\text{GS}\rangle = \frac{1}{\sqrt{2}} (|0000\rangle + |1111\rangle) \quad \langle \text{GS} | c^\dagger M c | \text{GS} \rangle = 0$$

This interaction preserves $\mathcal{G} = CT$, and a $SU(4)$ flavor symmetry, but it breaks $U(4)$.

In fact, we can prove that there could be a mixed anomaly between $\mathcal{G} = CT$ and flavor symmetry. The largest flavor symmetry free of mixed anomaly with $\mathcal{G} = CT$ is $\text{Spin}(7)$, meaning the 8 flavors of Majorana fermions form a spinor rep of $SO(7)$.

$SU(4)$ is a subgroup of $\text{Spin}(7)$, but not $U(4)$, therefore **SMG can happen with $SU(4)$ symmetry, but not with $U(4)$ flavor symmetry.**

Symmetries and Anomalies relevant to Symmetric Mass Generation

Mixed anomaly between $\mathcal{G}$ and flavor symmetry in 0+1d:

SMG can happen with SU(4) symmetry, but not with U(4) flavor symmetry.

Let's gauge the U(4) flavor symmetry, and break the CT symmetry with a mass term:

$$S[c, A] = \int d\tau \, c^\dagger (\partial_\tau + iA_\tau + m) c.$$

Integrating out the fermions will generate a 0+1d CS term:

$$S_{\text{CS}}[A] = ik \oint \text{Tr } A. \qquad k(m) = k_0 + \frac{1}{2} \text{sgn}(m).$$

If we want to keep CT , the gauge invariance is broken:

$$k(m > 0) = \frac{1}{2}, \qquad k(m < 0) = -\frac{1}{2}.$$

Symmetries and Anomalies relevant to Symmetric Mass Generation

Mixed anomaly between $\mathcal{G}$ and flavor symmetry in 0+1d:

SMG can happen with $SU(4)$ symmetry, but not with $U(4)$ flavor symmetry.

Let's gauge the $U(4)$ flavor symmetry, and break the CT symmetry with a mass term:

$$S[c, A] = \int d\tau \, c^\dagger (\partial_\tau + iA_\tau + m) c.$$

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$$S_{\text{CS}}[A] = ik \oint \text{Tr} A. \quad k(m > 0) = \frac{1}{2}, \quad k(m < 0) = -\frac{1}{2}.$$

$$U(4) = \frac{SU(4) \times U(1)}{\mathbb{Z}_4}.$$

$$S_{\text{CS}}[A] = ik \oint \text{Tr} A = 4ik \oint A^{U(1)}$$

At first glance, the level for the $U(1)$ gauge field is $4k$, which is an integer even for $k = 1/2$. But **$U(1)$ gauge flux fractionalizes, it takes integer times $2\pi/4$ here, hence anomaly.**

Symmetries and Anomalies relevant to Symmetric Mass Generation

Example of SMG in 0+1d:

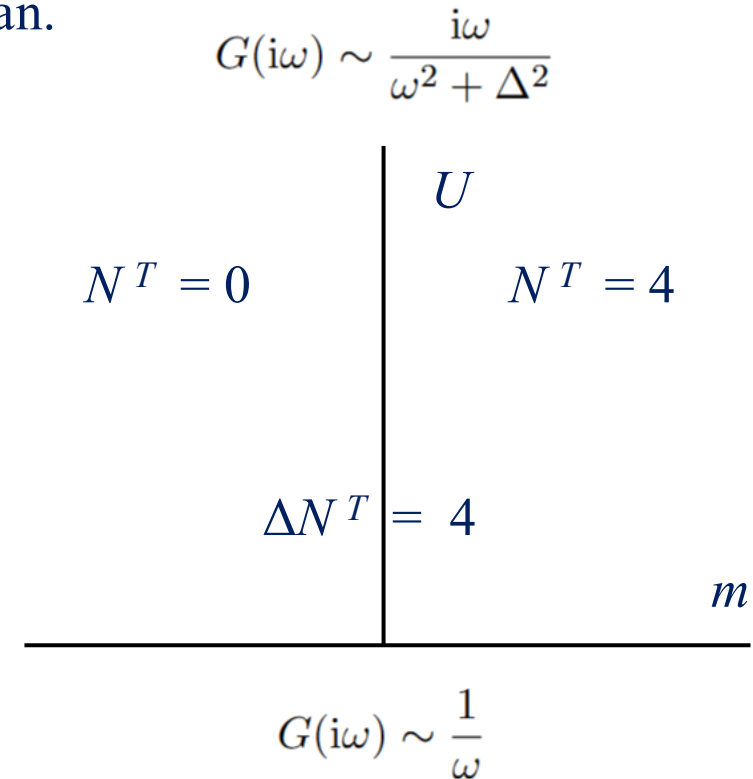
Now let us include a CT breaking mass term in the Hamiltonian.

$$H = \sum_{i=1}^4 m c_i^\dagger c_i - U (c_1 c_2 c_3 c_4 + h.c.)$$

Consider the fermion Green's function, and the following function

$$N^T = \frac{1}{2\pi i} \int_{-\infty}^{+\infty} d\omega \operatorname{tr} (G^{-1}(i\omega) \partial_\omega G(i\omega))$$

Guaranteed to be an integer if well defined, thanks to $\pi_1[\mathrm{GL}(N, C)]$. It can only change discontinuously when Green's functions become singular. But it can change through either “poles”, or “zeros” of the Green's function.



Symmetries and Anomalies relevant to Symmetric Mass Generation

Examples of SMG in 1+1d

Lieb-Shultz-Mattis theorem:

Simplest version: for a spin system with $SU(2)$ symmetry and translation symmetry, with odd number of spin-1/2s per unit cell, the system must be either gapless, or gapped but degenerate, as long as the Hamiltonian is local.

This is a statement at the level of UV lattice model.

LSM theorem in the UV implies SMG cannot happen in the IR analysis (must be some uncanceled 't Hooft anomalies of exact symmetries in the IR).

Symmetries and Anomalies relevant to Symmetric Mass Generation

Examples of SMG in 1+1d

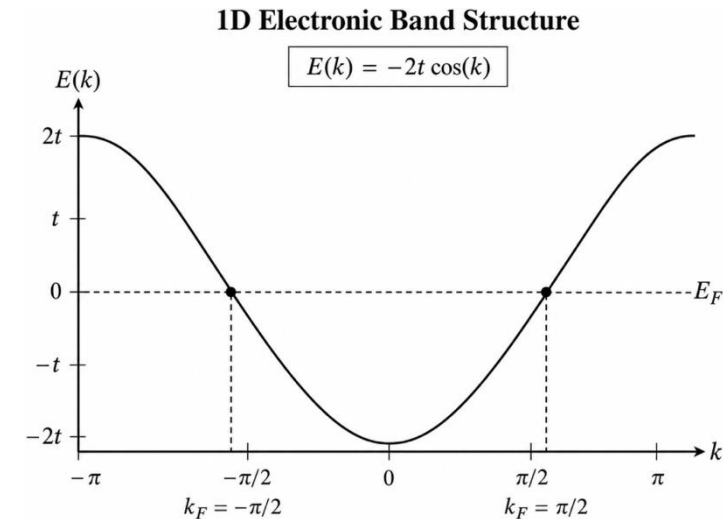
Example 1: 1+1d QED with $N_f=2$ Dirac fermions, and $SU(2)_V$ flavor symmetry. It is the $SU(2)_1$ CFT, the CFT describing the quantum spin-1/2 chain. Based on well-known result from Haldane, and also the **LSM theorem**, this system **cannot be trivially gapped (no SMG)**.

Spin-Z4: $x \rightarrow x + 1$, $\psi_R \rightarrow i\psi_R$, $\psi_L \rightarrow -i\psi_L$

Spin-Z4/Chiral-Z2 anomaly **not canceled**.

$$S = \int dx d\tau \sum_{a=1,2} \bar{\psi}_a \gamma_\mu (\partial_\mu - iA_\mu) \psi_a$$

$$H = \sum_i J_1 \vec{S}_i \cdot \vec{S}_{i+1} + \sum_i J_2 \vec{S}_i \cdot \vec{S}_{i+2}$$



Indeed, LSM theorem in the UV, consistent with the fact that some anomalies are not cancelled in the IR theory.

Symmetries and Anomalies relevant to Symmetric Mass Generation

Examples of SMG in 1+1d

Example 2: 1+1d QCD with U(2) gauge field, and $N_f = 2$ Dirac fermions, again with $SU(2)_V$. It is the $SU(2)_2$ CFT, the CFT describing a critical point of the quantum spin-1 chain. Based on well-known result from Haldane, this system **can be trivially gapped**.

Spin-Z₄: $x \rightarrow x + 1$, $\psi_R \rightarrow i\psi_R$, $\psi_L \rightarrow -i\psi_L$

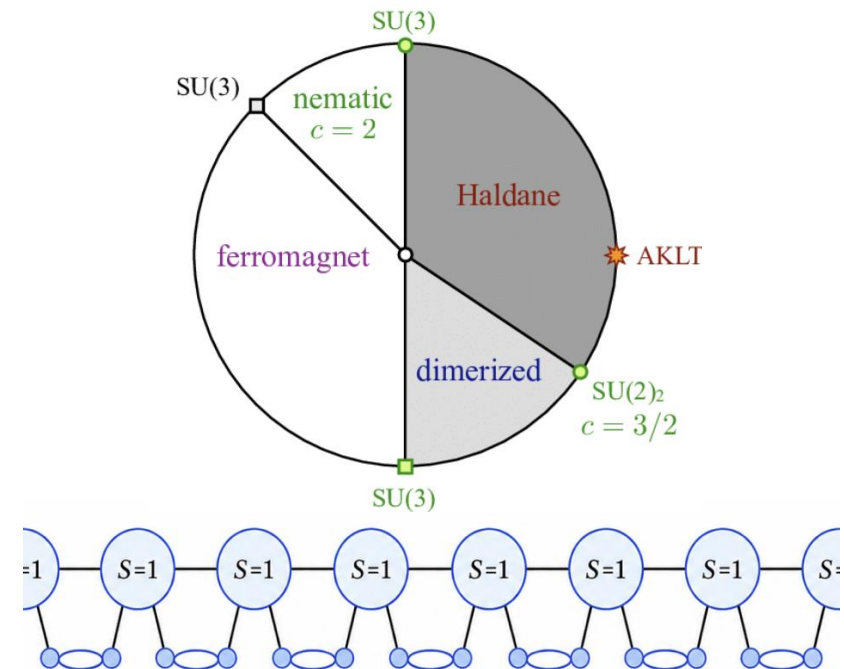
Spin-Z₄/Chiral-Z₂ anomaly **canceled**.

$$S = \int dx d\tau \sum_{a=1,2} \bar{\psi}_a \gamma_\mu (\partial_\mu - iA_\mu - i\vec{a}_\mu \cdot \vec{\sigma}/2) \psi_a$$

$$H = \sum_i J_1 \vec{S}_i \cdot \vec{S}_{i+1} + \sum_i J_2 (\vec{S}_i \cdot \vec{S}_{i+1})^2$$

The $SU(2)_2$ CFT has an emergent (enlarged) chiral flavor symmetry $G_{IR} = SU(2)_L \times SU(2)_R$. In SMG phase, this G_{IR} symmetry must be broken.

SU(2) spin-1 chains



Symmetries and Anomalies relevant to Symmetric Mass Generation

Examples of SMG in 1+1d

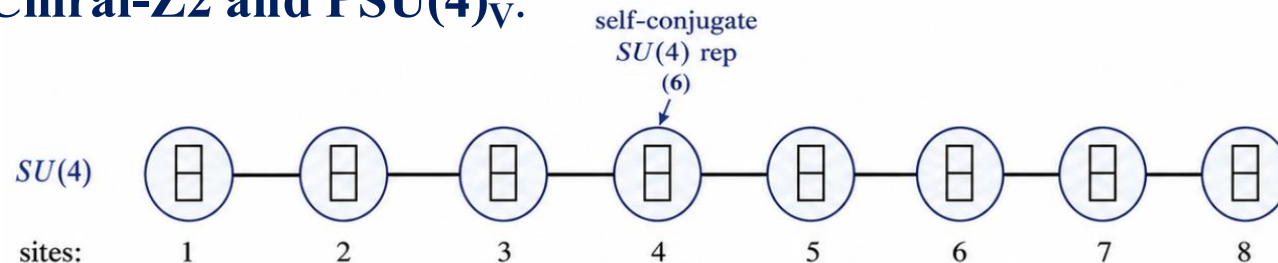
Not only need to cancel self-anomaly of Spin-Z₄, but also need to cancel the mixed anomaly between Spin-Z₄ and other (vector) flavor symmetries.

Example 3: 1+1d QED with $N_f = 4$ Dirac fermions, with $SU(4)_V$. It is the $SU(4)_1$ CFT, the CFT describing the quantum $SU(4)$ chain. If we also (only) assume the Spin-Z₄, this theory describes the $SU(4)$ chain with self-conjugate rep. This model **cannot be trivially gapped, due to LSM**.

Spin-Z₄/Chiral-Z₂ anomaly indeed **canceled**.

But some anomalies must remain uncanceled.
Indeed, in field theory, there is another **mixed anomaly between Chiral-Z₂ and $PSU(4)_V$** .

$$S = \int dx d\tau \sum_{a=1 \dots 4} \bar{\psi}_a \gamma_\mu (\partial_\mu - i A_\mu) \psi_a$$



Symmetries and Anomalies relevant to Symmetric Mass Generation

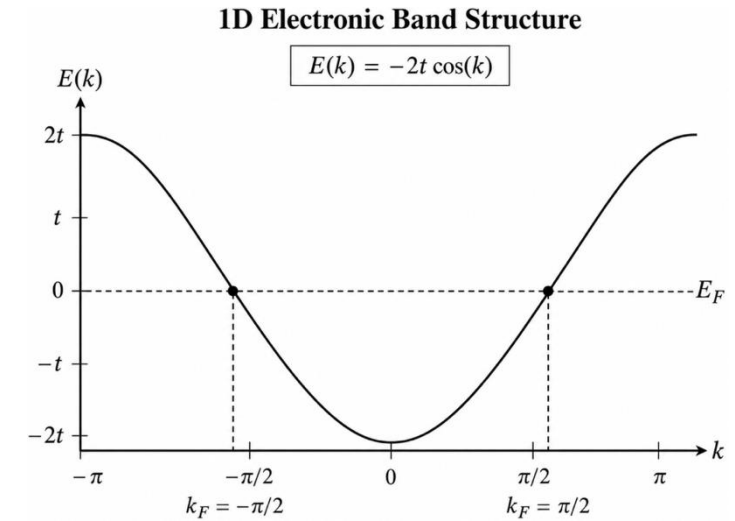
Examples of SMG in 1+1d

Now consider a bilayer (two-orbital) spin-1/2 quantum Hubbard model in 1d.

$$H = \sum_{i,j} \sum_{a=1,2} \sum_{l=1,2} -t c_{i,a,l}^\dagger c_{j,a,l} + h.c. + H_{\text{int}}$$

The noninteracting part of the system maps to Dirac fermion in the same way as before, as Spin-Z₄ implemented as translation. But now there are $N_f = 4$ Dirac fermions. The Spin-Z₄ anomaly is cancelled out, and there is no LSM obstruction. Indeed, **the system can be drive into a trivially gapped (SMG) state (Slagle et.al. 1409.7401).**

The vector flavor symmetry $U(4)_V$ has a mixed 't Hooft anomaly with Spin-Z₄, therefore there cannot be SMG phase with $U(4)$ symmetry, consistent with LSM theorem.



Symmetries and Anomalies relevant to Symmetric Mass Generation

Example of SMG in 2+1d

Key symmetry is a nonunitary $\mathcal{G} = CT$. One can verify that, with CT , all Dirac fermion mass operators change sign:

$$\bar{\psi} M \psi \rightarrow -\bar{\psi} M \psi.$$

The self 't Hooft anomaly of CT is cancelled for multiple of 8 Dirac fermions, or 16 Majorana fermions (famous Z_{16} classification of ${}^3\text{He-B}$).

But there could still be mixed anomaly between $\mathcal{G} = CT$ and flavor symmetry. For example, for 8 flavors of Dirac fermions, if there is a $SU(8)$ flavor symmetry, **there is a mixed anomaly between $SU(8)$ and $\mathcal{G} = CT$.**

If we gauge $SU(8)$, there will be a parity anomaly. $CT + \text{Gauge } SU(8) \rightarrow 1/2 \text{ CS}(A)$

Flavor symmetries with mixed anomaly with $\mathcal{G} = CT$: $SO(16)$, $SU(8)$, $Sp(4)$...

Flavor symmetries without mixed anomaly with $\mathcal{G} = CT$: $Spin(7)$, $SU(4)$...

Symmetries and Anomalies relevant to Symmetric Mass Generation

Example of SMG in 2+1d

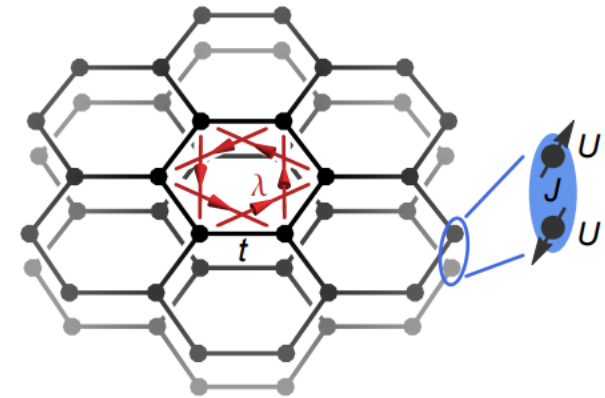
Key symmetry is a nonunitary $\mathcal{G} = CT$. One can verify that, with CT , all Dirac fermion mass operators change sign:

$$\bar{\psi} M \psi \rightarrow -\bar{\psi} M \psi.$$

For a 2d (or 2+1d) lattice model, this symmetry must be emergent (for example, $C2 \times PH$).

Different UV versions, similar results:

Slagle et.al. 1409.7401 Bilayer spin-1/2 quantum Hubbard model on 2d honeycomb. Similar version, He, et.al. 1603.08376, Euclidean spacetime version: Ayyar, Chandrasekharan, 1410.6474, Catterall, 1510.04153, Maiti et.al. 2602.18360,



Current numerics support a continuous transition between massless Dirac fermion phase to the SMG phase.

Symmetries and Anomalies relevant to Symmetric Mass Generation

Conditions for SMG in 3+1d (4d) lattice-QCD

Let's take staggered fermion as a potential platform for SMG. One staggered fermion is $N_f = 4$ QCD in the IR.

Does staggered fermion have the key symmetry $\mathcal{G} = \text{Spin-Z}_4$?

Exact symmetries of staggered fermion, includes the $U(1)_\epsilon$, and “shift” symmetries (translation by one lattice constant along each direction)

$$\begin{aligned} \text{Spin-Z}_4 : \psi &\rightarrow e^{ik\frac{\pi}{2}\gamma_5}\psi, & \text{is a subgroup of} & & U(1)_\epsilon : \psi &\rightarrow e^{i\theta\gamma_5\xi_5}\psi. \\ & & & & Z_2^\xi : \psi &\rightarrow \xi_5\psi. \end{aligned}$$

Z_2^ξ is the Z_2 subgroup of shift symmetry, which translates along all four directions by one lattice constant.

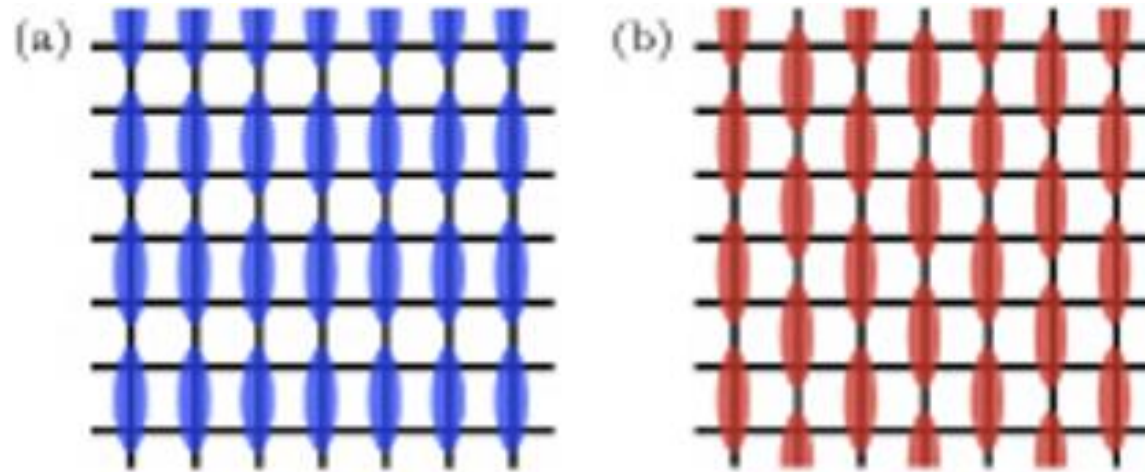
Therefore it is important to keep Z_2^ξ (not necessarily the entire shift symmetry) for SMG,

Symmetries and Anomalies relevant to Symmetric Mass Generation

Conditions for SMG in 3+1d (4d) lattice-QCD

Let's take staggered fermion as a potential platform for SMG. One staggered fermion is $N_f = 4$ QCD in the IR.

To verify SMG, need to check whether $\mathcal{G} = \text{Spin-Z}_4$ is spontaneously broken



Break Z_2^ξ and Spin-Z₄,
A fermion mass term

Preserve Z_2^ξ and Spin-Z₄,
not a mass term

Symmetries and Anomalies relevant to Symmetric Mass Generation

Necessary conditions for SMG to happen in a 3+1d (or 4d) lattice-QCD model

- (1) $N_f N_c$ is multiple of 8, so Spin-Z4 anomaly is cancelled
- (2) $N_f N_c$ there is no mixed anomaly between vector symmetry G_V and Spin-Z4.

For example, for $N_f = 8$, $N_c = 3$, even though the Spin-Z4 self-anomaly is cancelled, there is a mixed anomaly between $SU(8)_V$ and Spin-Z4. This case can be easily diagnosed:

We gauge $SU(8)_V$, Chiral-Z2 is broken through ABJ, **meaning $SU(8)_V$ and Spin-Z4 are incompatible**. Meaning, if SMG happens, the **$SU(8)_V$ must be broken**.

$$\exp \left(i 2 \alpha N_c \int d^4 x \frac{1}{8 \pi^2} \text{tr} (F_f \wedge F_f) \right)$$

When N_f and N_c are not coprime, analysis is more subtle. For example, For $N_f = 4$, $N_c = 2$, there is still a **mixed anomaly between $U(4)/Z2$ and Spin-Z4**.

Related analysis in other context: Tanizaki, 2018, Cordova, Freed, Lam, Seiberg, 2019, etc.

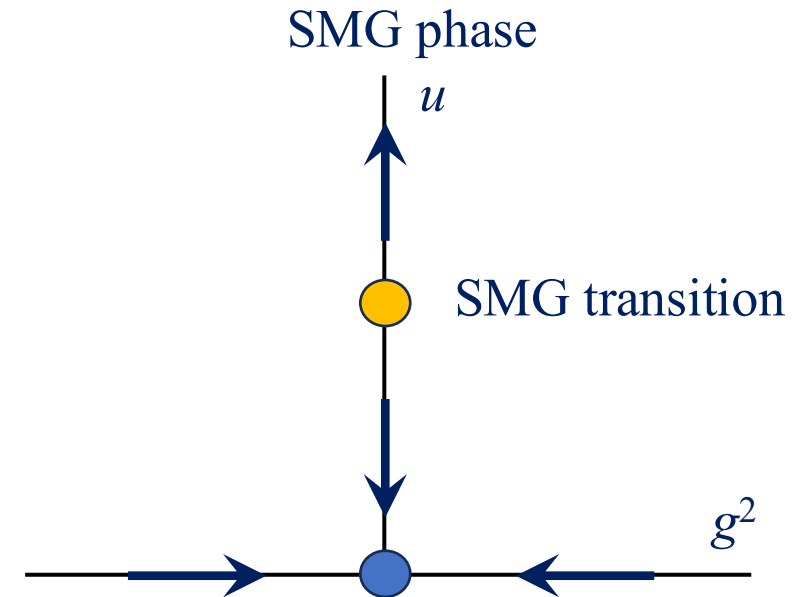
Symmetries and Anomalies relevant to Symmetric Mass Generation

Necessary conditions for SMG to happen in a 3+1d (or 4d) lattice-QCD model

If SMG happens in certain lattice-QCD model, besides the gauge coupling, we also need terms that break the vector symmetry with mixed anomaly with Spin-Z4.

Example of such terms: high-dimension terms for staggered fermion in Lee&Sharpe 1999.

Term in action	[Flavor] $\times$ rotation symmetry
S_4 ($m=0$)	$[SU(4)_L \times SU(4)_R] \times SO(4)$
S_4 ($m \neq 0$)	$[SU(4)] \times SO(4)$
S_6^{glue}	$[SU(4)_L \times SU(4)_R] \times SW_4$
S_6^{bilin} ($m=0$)	$[SU(4)_L \times SU(4)_R] \times SW_4$
S_6^{bilin} ($m \neq 0$)	$[SU(4)] \times SW_4$
$S_6^{\text{FF(A)}}$	$[U(1)_A \times \Gamma_4 \rtimes SO(4)] \times SO(4)$
$S_6^{\text{FF(B)}}$	$U(1)_A \times (\Gamma_4 \rtimes SW_{4,\text{diag}})$
S_6 ($m=0$)	$U(1)_A \times (\Gamma_4 \rtimes SW_{4,\text{diag}})$
S_6 ($m \neq 0$)	$\Gamma_4 \rtimes SW_{4,\text{diag}}$



Symmetries and Anomalies relevant to Symmetric Mass Generation

Procedures of looking for SMG:

1. If the system has a gapless point/phase described by massless fermions, identify the key symmetry $\mathcal{G}$ under which all fermion mass terms transform nontrivially, this symmetry must be exact, but could realize non-onsite.
2. Ensure that $\mathcal{G}$ has no 't Hooft anomaly, including **its self 't Hooft anomaly, as well as mixed anomaly with other symmetries.**
3. In the putative trivially gapped (SMG) phase, verify that the key symmetry $\mathcal{G}$ is preserved.
4. It would be best if there is a continuous transition between SMG and the gapless phase.
5. Check other phenomena of the SMG phase, like the Green's function zero.

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