Vacuum degeneracies and solitons in 2D QCD theories

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Princeton University

Based on [2601.16262], [2508.16363] with Ross Dempsey, Anna-Maria Glück, and Silviu Pufu
and WIP also including Clay Cordova, Nicholas Holfester

Syracuse, August 14th, 2026

Introduction

- Recent developments in methods for constraining the dynamics of gapped $(1+1)$d gauge theories [Komargodski, Ohmori, Roumpedakis, Seifnashri 2020; Delmastro, Gomis, Yu 2021; Delmastro, Gomis 2022; Damia, Galati, Tizzano 2024; Cordova, García-Sepúlveda, Holfester 2024; ...]
- Gapped QCD-like theory typically has many degenerate vacua that are mapped into each other by (non-)invertible symmetries
- Challenge: Known lattice models break non-invertible symmetries
  $\Rightarrow$ **states mix in finite volume**
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  $\Rightarrow$ can study all vacua using uniform matrix product states and link-enhanced matrix product operators [Dempsey, Glück, Pufu, BTS 2025]

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Continuum model

- $(1+1)$d Yang-Mills theory with gauge group G coupled to a **massless** Majorana fermion in a real representation $\boldsymbol{\lambda}$

$$S = \int d^2x \left(-\frac{1}{4g^2} F_A^{\mu\nu} F_{\mu\nu}^A + \frac{i}{2} \bar{\psi}_a \gamma^\mu (D_\mu \psi)_a \right)$$

- Generically the IR is a CFT, but can be gapped [Delmastro, Gomis, Yu 2021]
- Examples of gapped theories:
  - SU(N) adjoint QCD (SU(N) $+ \psi_{\text{adj}}$) [Gross, Klebanov, Matytsin, Smilga 1996; Cherman, Jacobson, Tanizaki, Ünsal 2019; Komargodski, Ohmori, Roumpedakis, Seifnashri 2020; Dempsey, Klebanov, Pufu, BTS 2023; ...]
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$SU(2) + \psi_{\mathbf{5}}$: Vacuum structure

- $(1+1)$d gauge coupling is relevant $\Rightarrow$ deep IR dynamics governed by a topological coset ($c = 0$) with 4 states/vacua

$$\mathrm{SU}(2) + \psi_{\mathbf{5}} \quad \overset{\mathrm{IR}}{\rightsquigarrow} \quad \frac{\mathrm{SO}(5)_1}{\mathrm{SU}(2)_{10}}$$

- Internal invertible symmetries:
  - $\mathbb{Z}_2^{[1]}$ center symmetry: splits Hilbert space into flux tube sectors (like discrete θ-term)
  - $(\mathbb{Z}_2)_\chi$ chiral rotation: $\psi_a \to \gamma^5 \psi_a \Rightarrow$ vanishing string tension
  - $(\mathbb{Z}_2)_F$ fermion parity: $\psi_a \to -\psi_a$
- The vacuum degeneracy cannot be explained by invertible symmetries

$$\begin{tikzcd}[column sep=huge]
\underline{p=0} & \underline{p=1} \\
\blacksquare \arrow[r, leftrightarrow, "(\mathbb{Z}_2)_\chi"] \arrow[d, phantom, "\text{non-inv. } \wr"] & \blacksquare \arrow[d, phantom, "\wr \text{ non-inv.}"] \\
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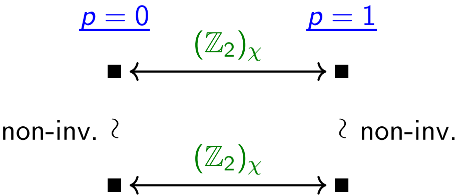

Lattice Hamiltonian for QCD theories

- Gauge fields: parallel propagator U^{ab}_n, electric field L^A_n, R^A_n
  Matter: staggered Majorana fermions χ^a_n with $\{\chi^a_n, \chi^b_m\} = \delta^{ab}\delta_{nm}$
  [Dempsey, Klebanov, Pufu BTS 2023]

$A, B, \ldots =$ adjoint indices
$a, b, \ldots =$ **5** indices

$$H = \sum_n \left[\frac{ag^2}{2} L^A_n L^A_n - \frac{i}{2a} \chi^a_n U^{ab}_n \chi^b_{n+1} \right]$$

- Gauss law constraint

$$L^A_n - R^A_{n-1} = Q^A_n = \frac{1}{2} \chi^a_n T^A_{ab} \chi^b_n$$

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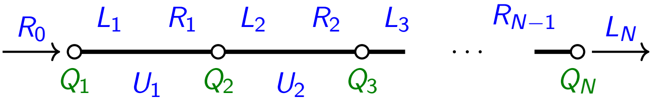

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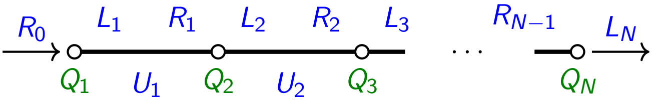

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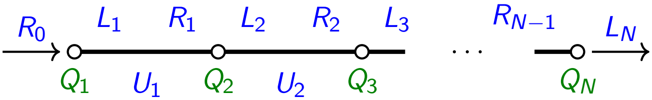

Lattice Hilbert space for SU(2) $+ \psi_5$

- Local basis states: [See also Shailesh's talk]
  - Gauge fields: labeled by electric field strength L_n^A i.e. irreps of SU(2) $\boldsymbol{r}_n$ (and (m, m')-indices)

$$|\boldsymbol{r}_1; m_1, m_1'\rangle \dots |\boldsymbol{r}_n; m_n, m_n'\rangle \dots |\boldsymbol{r}_{N-1}; m_{N-1}, m_{N-1}'\rangle$$

  - Matter fields: a real fermion χ_n^a with a **5**-index generate the irrep **4** (and an uncharged Majorana λ_n)

$$|\mathbf{4}; m_1\rangle \dots |\mathbf{4}; m_n\rangle \dots |\mathbf{4}; m_N\rangle \, |k\rangle \ , \qquad k = 1 \dots 2^{N/2}$$

- In this talk: focus on color charge

$|\{\boldsymbol{r}_n\}; \text{other QNs}\rangle =$

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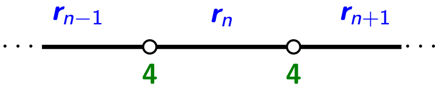
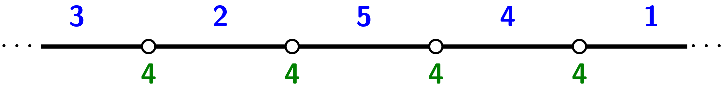


Physical states

- The Gauss law constraint is imposed by: $\boxed{\mathbf{r}_n \subset \mathbf{4} \otimes \mathbf{r}_{n-1}}$ (and contracting with Clebsch-Gordan coeffs)
- An example of a physical state:

$|\psi_{\text{phys}}\rangle =$ 

- Translational invariant states: $|\{\mathbf{r}_1, \mathbf{r}_2\}\rangle$ with $\mathbf{r}_2 \subset \mathbf{4} \otimes \mathbf{r}_1$ and $\mathbf{r}_1 \subset \mathbf{4} \otimes \mathbf{r}_2$

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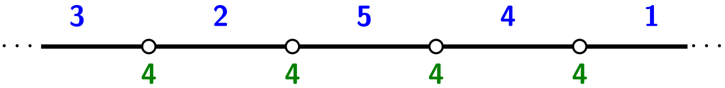
$|\psi_{\text{phys}}\rangle =$ 

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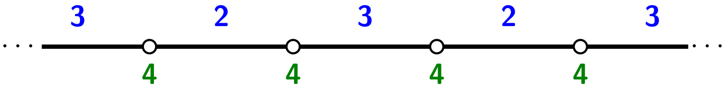
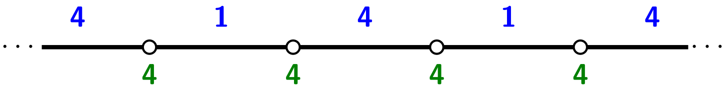
$|\{\mathbf{3}, \mathbf{2}\}\rangle =$ 

$|\{\mathbf{4}, \mathbf{1}\}\rangle =$ 

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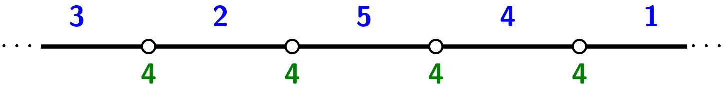
$|\psi_{\text{phys}}\rangle =$ 

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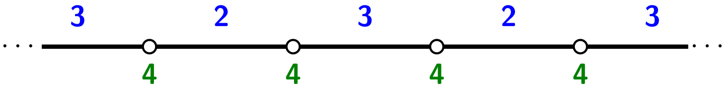
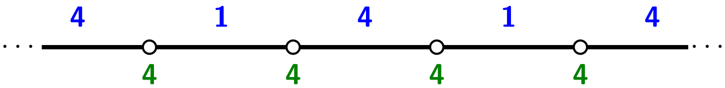
$|\{\mathbf{3}, \mathbf{2}\}\rangle =$ 

$|\{\mathbf{4}, \mathbf{1}\}\rangle =$ 

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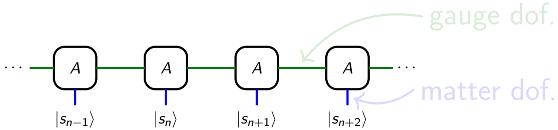
Uniform MPS and link operators

- uMPSs are variational ansätze for gapped ground states **directly on an infinite lattice**, parameterized by A^s_{ij} with $d \times \chi \times \chi$ components

$$|\psi\rangle = \sum_{\{i_n,s_n\}} \prod_n A^{s_n}_{i_{n-1}i_n} |s_n\rangle = \cdots$$



- Symmetric MPSs naturally encode gauge theory states [See Ross's talk; Dempsey, Glück, Pufu, BTS 2025]
- Electric field acts on **links** and allow for a **local** expression

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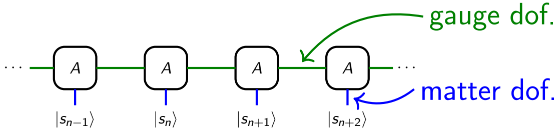

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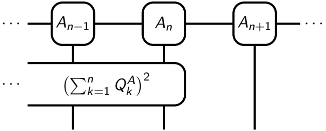

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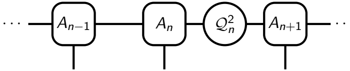

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Cluster decomposition and local minima

- Would-be vacua are distinguished by satisfying cluster decomposition
- A generic uMPS satisfies cluster decomposition

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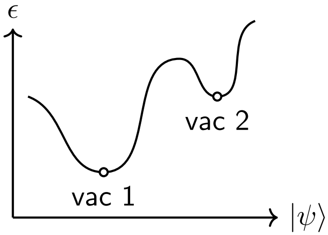

Vacuum degeneracy: SU(2) + ψ_5

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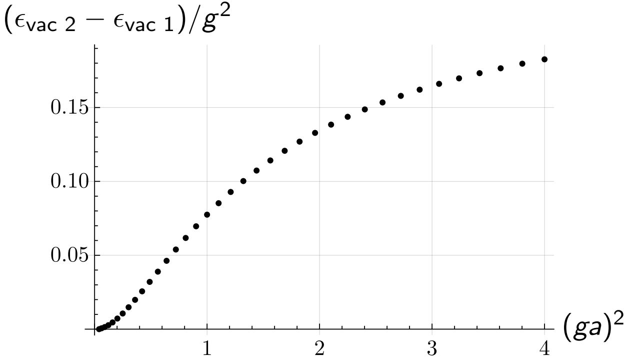
- Recall: continuum has 2 vacua in each flux tube sector



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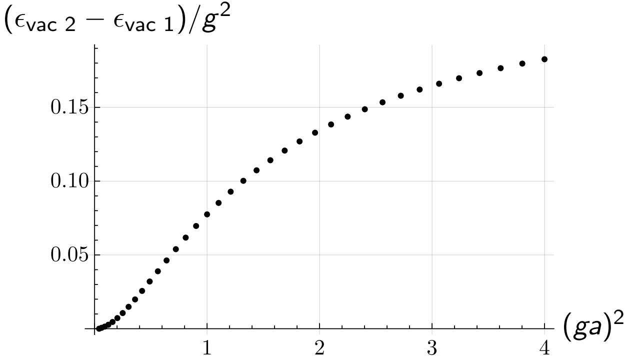

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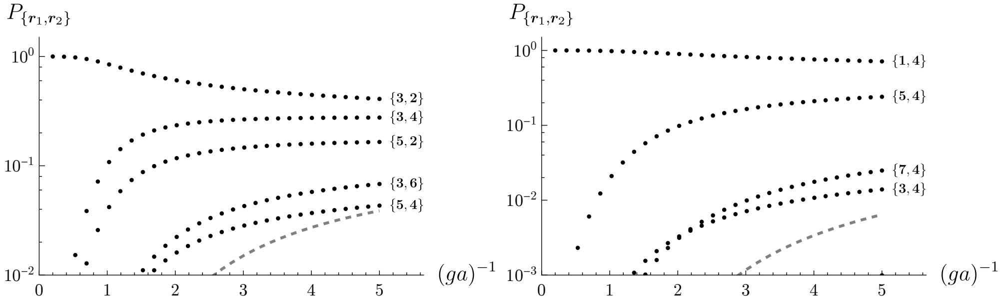
Strong coupling limit of continuum vacua

- The continuum vacua are continuously connected to strong coupling states labeled by electric field strengths

$\boxed{\text{vac 1}}$: $\ldots, \mathbf{3}, \mathbf{2}, \mathbf{3}, \mathbf{2}, \ldots$, $\boxed{\text{vac 2}}$: $\ldots, \mathbf{1}, \mathbf{4}, \mathbf{1}, \mathbf{4}, \ldots$



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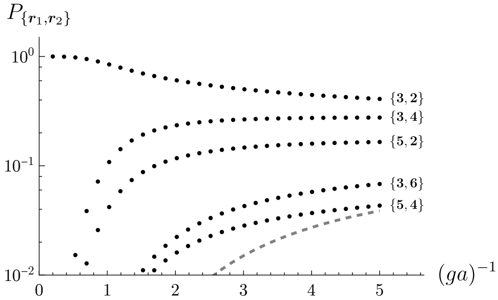

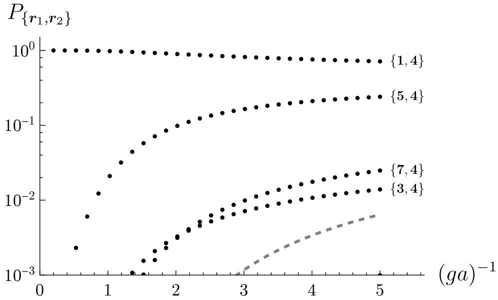

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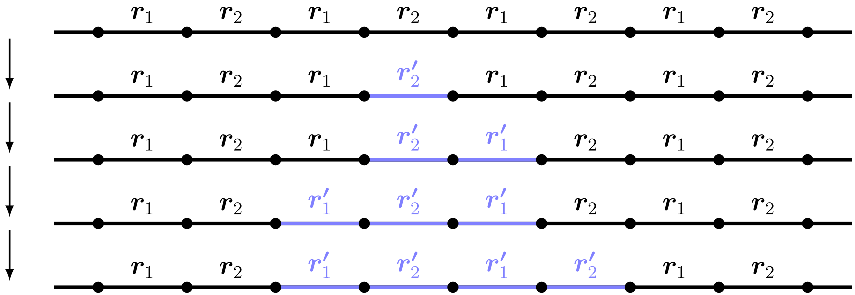
Lattice decay rule

- Minimize strong coupling Hamiltonian + satisfy the Gauss law

$$H \xrightarrow{ag\to\infty} \frac{ag^2}{2}\sum_n L_n^A L_n^A\,, \qquad \boldsymbol{r}_n \subset \mathbf{4}\otimes \boldsymbol{r}_{n-1}$$

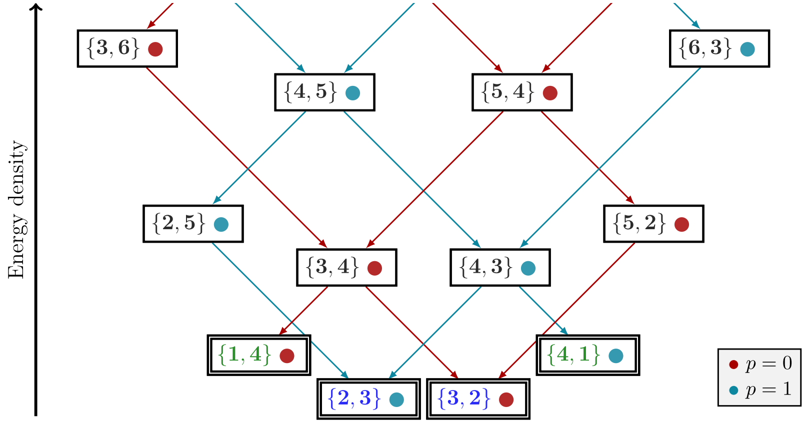
- Bubble nucleation: lower the Casimir one link at a time respecting the Gauss law



- Lattice decay rule: follow decay chain of translation invariant states $|\{\boldsymbol{r}_1, \boldsymbol{r}_2\}\rangle$

Lattice decay rule: SU(2) + ψ_5

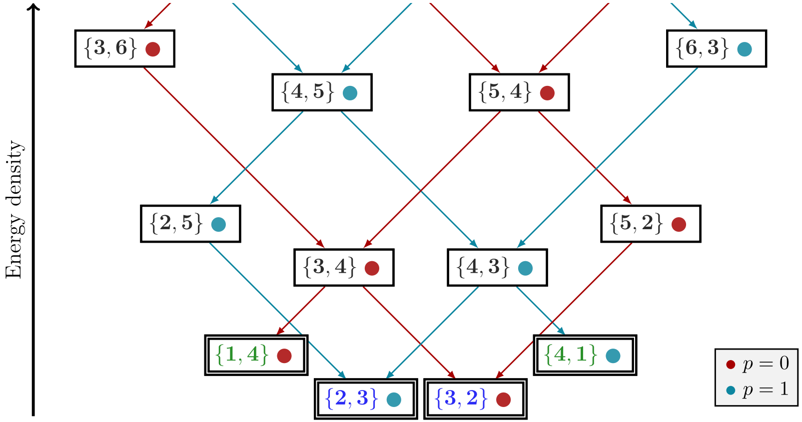
- The lattice decay rule identifies the strong coupling state for all 4 continuum vacua



- Agrees with coset calculation. Checked in many more examples e.g. USp(8) + ψ_{42} w/ **24 vacua**!

Lattice decay rule: SU(2) + $\psi_{\mathbf{5}}$

- The lattice decay rule identifies the strong coupling state for all 4 continuum vacua



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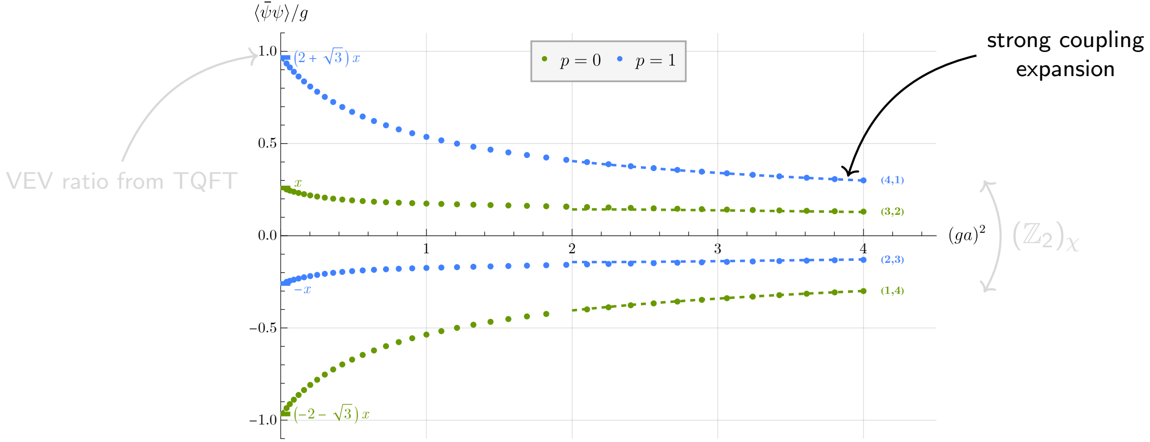
Chiral condensate: SU(2) + ψ_5

- Numerics: initialize ground state search with lattice decay rule
- Vacuum expectation values agree with TQFT prediction [Cordova, García-Sepúlveda, Holfester 2024]



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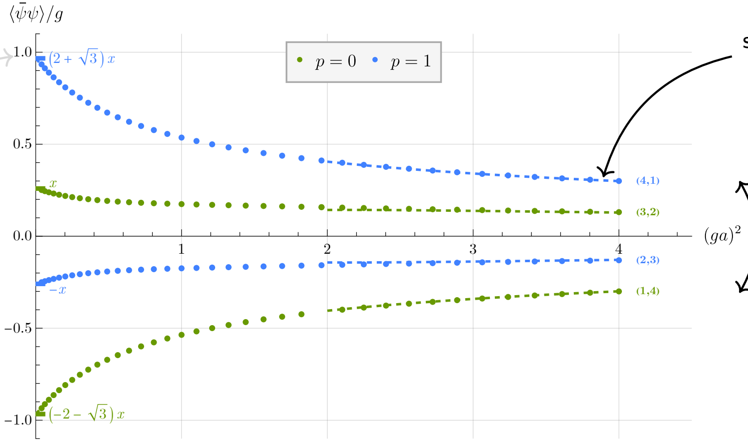
VEV ratio from TQFT



strong coupling expansion

$(\mathbb{Z}_2)_\chi$

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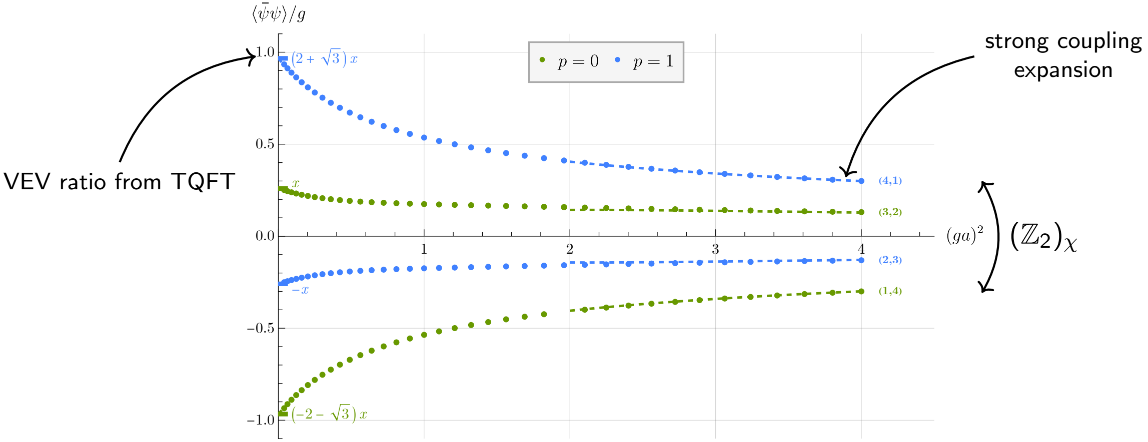
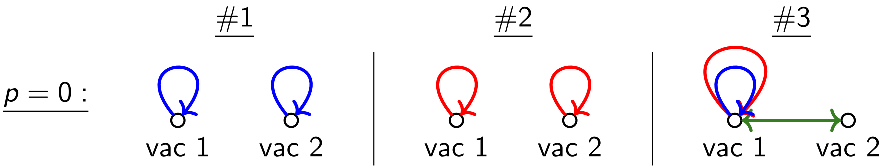


Excitations of SU(2) + $\psi_{\mathbf{5}}$

- Non-invertible symmetries give rise to multiplets of bosons, fermions, and solitons [Cordova, García-Sepúlveda, Holfester 2024; Cordova, Dempsey, Glück, Holfester, Pufu, BTS WIP]



- MPS quasiparticle ansatz: Stable **particles** as local excitations of the ground state [Haegeman, Osborne, Verstraete 2013]

$$|B, k\rangle = \sum_n e^{ikn}$$

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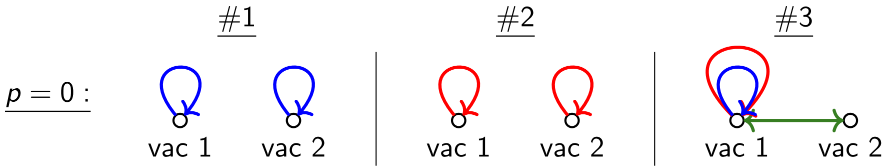

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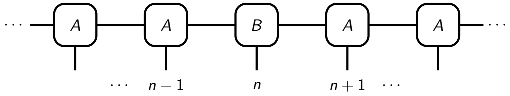
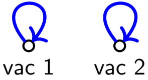
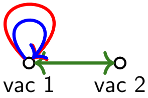
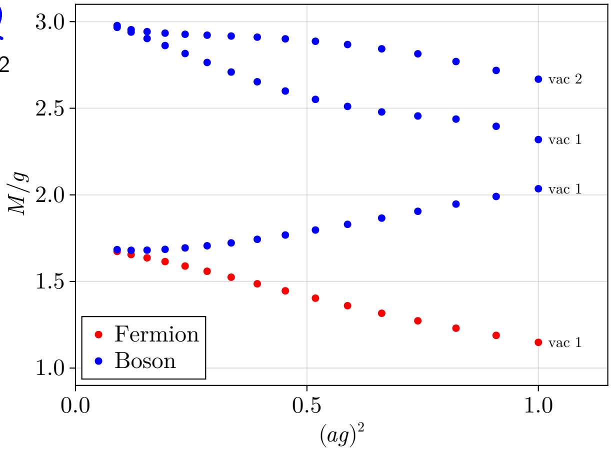


Particle excitations of SU(2) + ψ_5

- Quasiparticle ansatz finds multiplets #3 and #1 in the continuum limit $ag \to 0$
- Can we complete the #3 multiplet?







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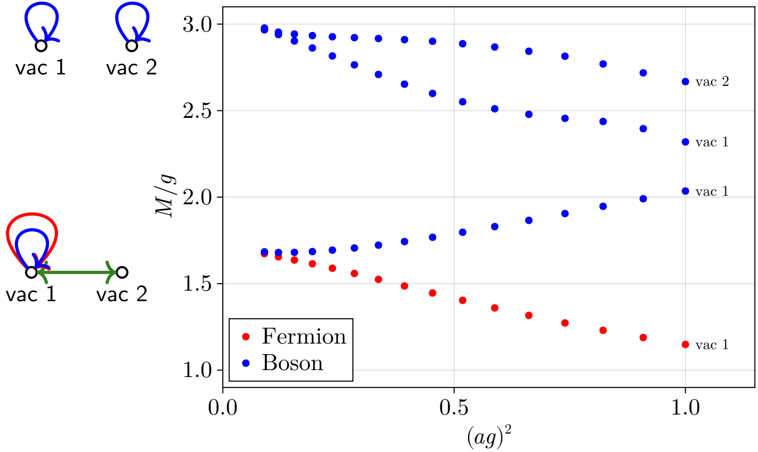
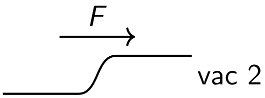


(Un)stable lattice solitons

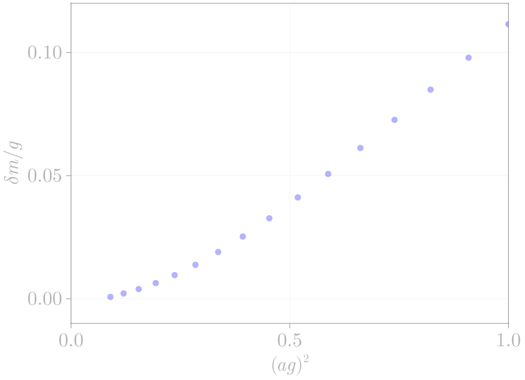
- At finite *ag* the vacua are **non-degenerate** and stable solitons do not exist



for $\epsilon_2 > \epsilon_1$

- Deform by mass term to get degeneracy at finite *ag* (chiral symmetry is broken)

$$H \longrightarrow H - \frac{i\delta m}{2}\sum_n (-1)^n \chi_n^a U_n^{ab} \chi_{n+1}^b$$



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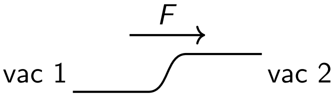

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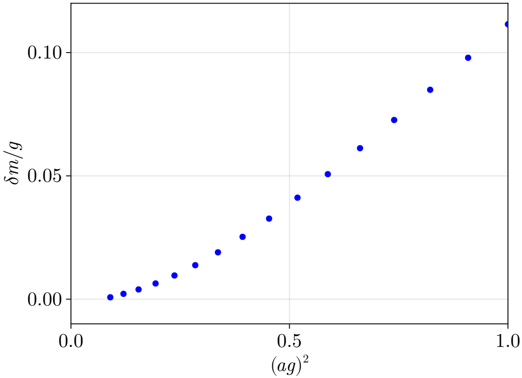

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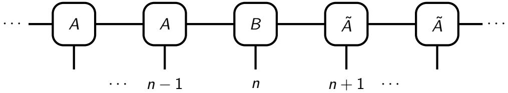
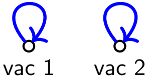
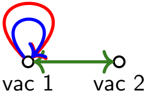
Particle/soliton degeneracy in SU(2) + ψ_5

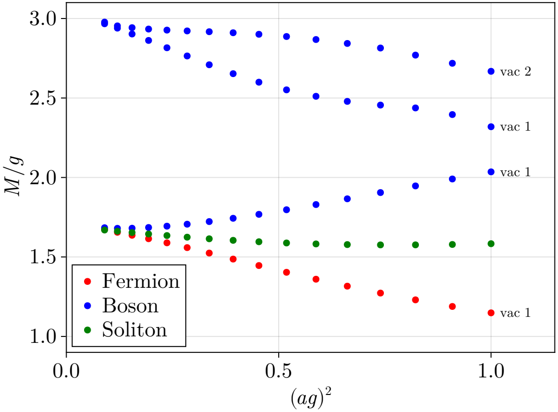
- Solitonic quasiparticle ansatz allows us to complete the <u>#3</u> multiplet

$$|B, k\rangle = \sum_n e^{ikn}$$









Summary

- We can study **all** vacua of gapped $(1+1)$d gauge theories using uniform MPS despite explicitly broken (non-invertible) symmetries
- The degenerate continuum vacua are continuously connected to non-degenerate strong coupling states
- The lattice decay rule determines all would-be vacua $\Rightarrow$ surprising link to topological cosets
- Numerical confirmation of particle/soliton degeneracy
- Future directions:
  - Universality of the multiplets appearing in gapped QCD theories
  - Construct lattice Hamiltonian with the non-invertible symmetry of the continuum.

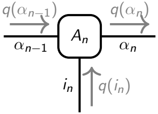
Extra Slides

Symmetric matrix product states

- Global **on-site** symmetry $U = \prod_n U_n \Rightarrow$ local constraints on MPS tensors
  [Sanz, Wolf, Perez-Garcia, Cirac 2009]

$$\left[(\mathcal{Q}^a_{n-1})_{\alpha_{n-1}\alpha'_{n-1}} + (Q_n)^{i_n i'_n} - (\mathcal{Q}_n)_{\alpha_n \alpha'_n}\right](A_n)^{i'_n}_{\alpha'_{n-1}\alpha'_n} = 0\,.$$

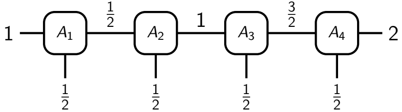
- Keep track of **virtual irreps** e.g. for U(1) symmetry



$$= 0 \qquad \text{if} \qquad q(\alpha_{n-1}) + q(i_n) \neq q(\alpha_n)\,.$$

In general, A_n can be non-zero when $\boldsymbol{r}_n \subset \boldsymbol{R}_n \otimes \boldsymbol{r}_{n-1}$

- SU(2) example of symmetric (matrix) product state:



A symmetric MPS includes sum over **fusion channels** and **multiplicities**