A new mass generation paradigm

Confinement and mass generation without Goldstone bosons

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University of Colorado Boulder

New developments on Lattice fermions”
Syracuse, Aug 12, 2026

Numerical work

- AH, *PRD* 106 (2022) 1, 014513
- AH, Witzel (LSD),2412.10322
- Butt, Catterall, AH,PRL 134 (2025) 3, 031602

Interpretation as continuum QFT

- AH, Cenke Xu, arXiv:2604.02424

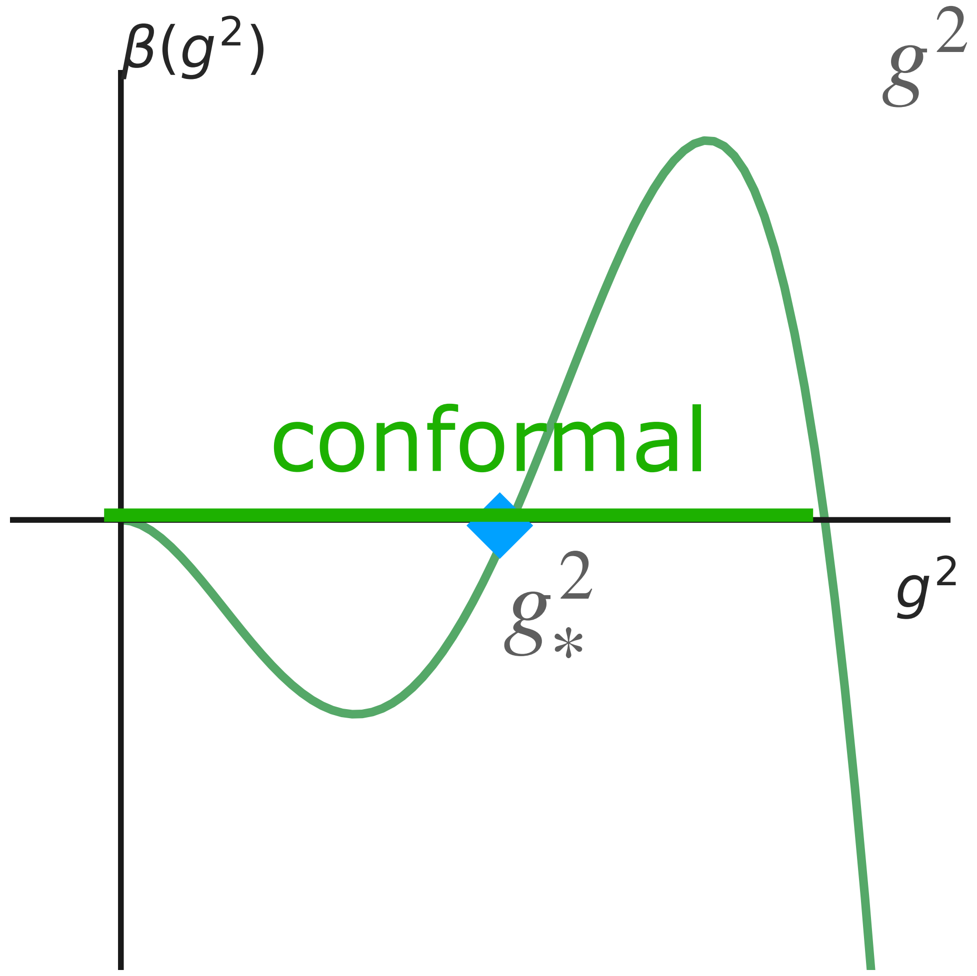
Massless gauge-fermion systems - the convention

Consider $SU(N_c)$ gauge theory with N_f massless fermions:

UV: $\tilde{G}_f = SU(N_f)_L \times SU(N_f)_R \times U(1)_V \times U(1)_A$

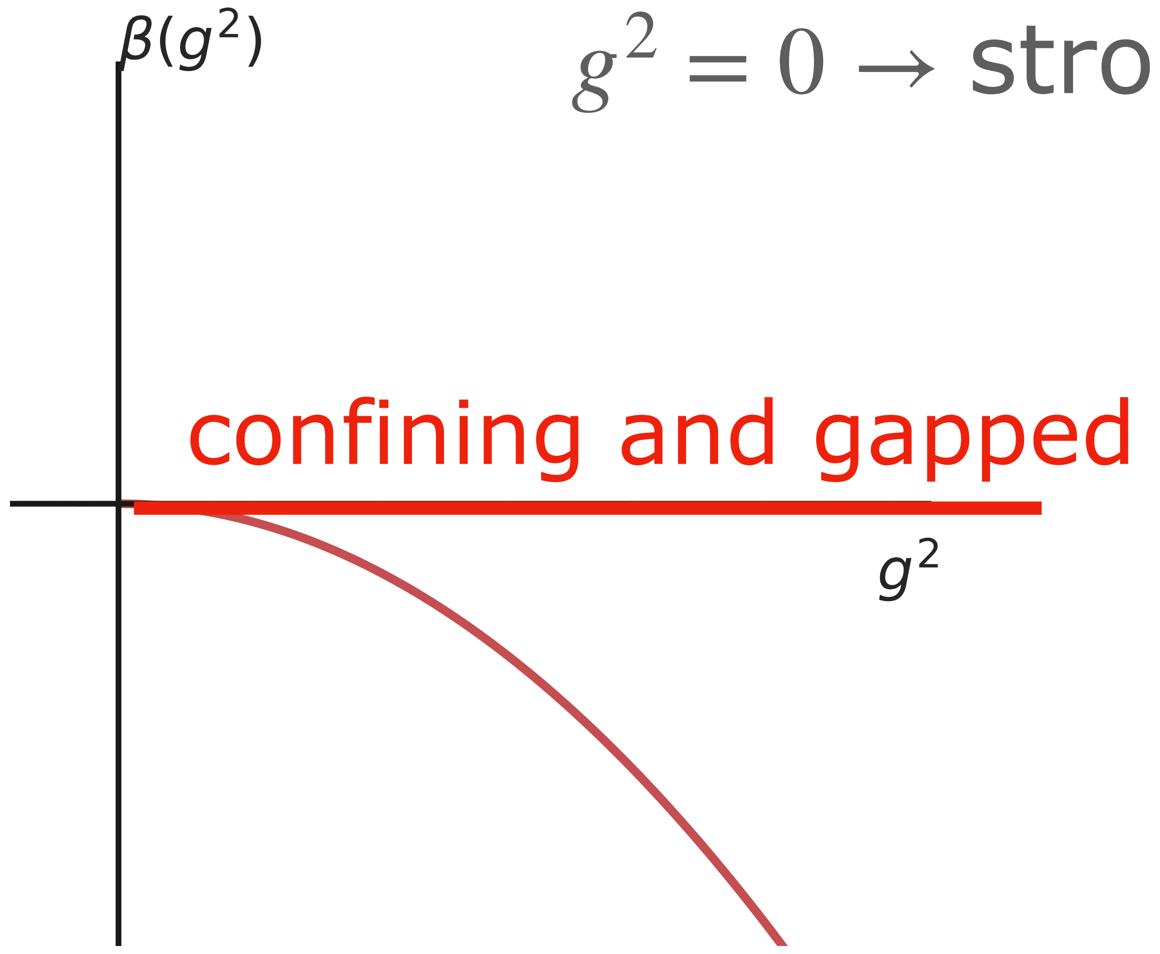
IR: two possibilities:

$g^2 = 0 \to g_*^2$



Conformal:
- asymptotically free
- gapless
- IRFP at g_*^2

$g^2 = 0 \to$ strong coupling



Spontaneous chiral symmetry breaking:
- asymptotically free
- confining
- $\langle \bar{\psi}\psi \rangle \neq 0$, Goldstones

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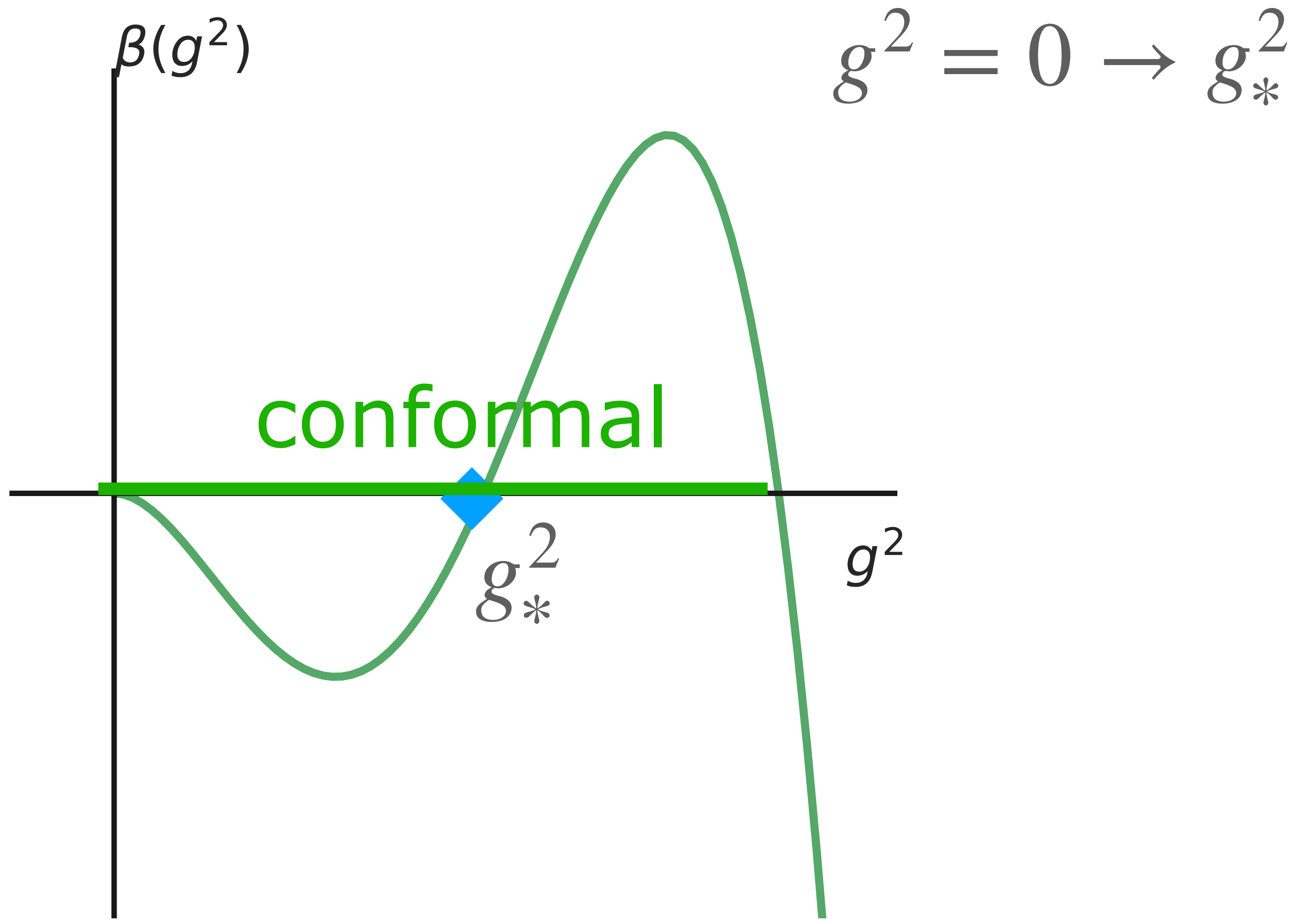

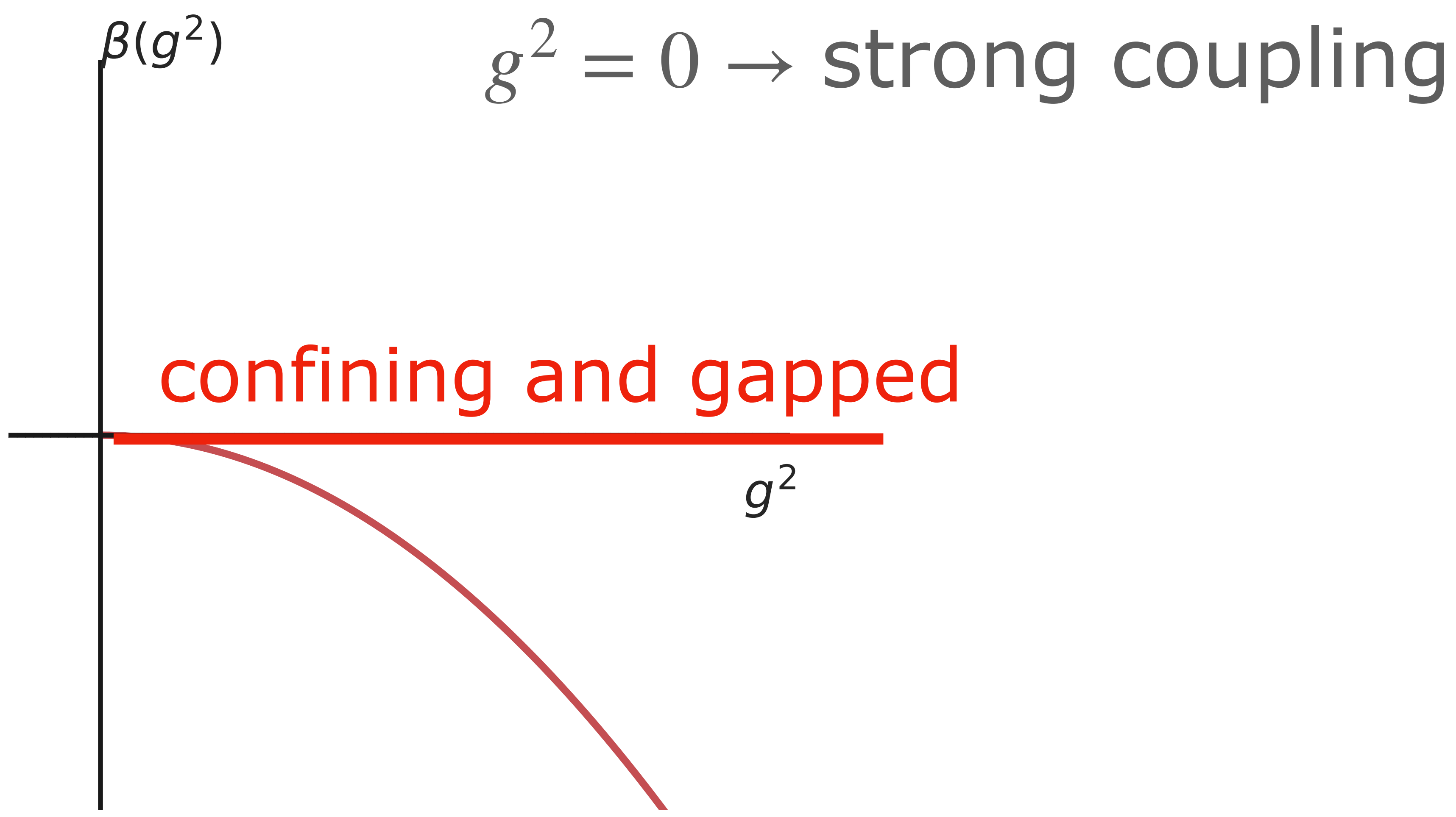

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Can there be a third possibility: confining + gapped +no Goldstone bosons ?
likely not asymptotically free

Anomalies determine what is allowed

AH, Cenke Xu, arXiv:2604.02424

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$U(1)_A$: ABJ anomaly $\longrightarrow$ not an exact symmetry

$SU(N_f)_L \times SU(N_f)_R$: has ’t Hooft anomalies $\longrightarrow$ no symmetric gapped IR

Explicitly reduce the anomalous chiral symmetry
$\Longrightarrow$ a symmetric gapped phase is allowed

If new interactions break

$U(1)_A \rightarrow \mathrm{Spin} - Z_4$ - the anomaly cancels **if** $N_f N_c = 0 \bmod 8$

$SU(N_f)_V \times SU(N_f)_A \rightarrow G_{\mathrm{allowed}}$ - non-anomalous

Two minimal gauge theories

Assume $U(1)_A \quad \rightarrow \quad \mathrm{Spin} - Z_4 \;,\; SU(N_f)_V \times SU(N_f)_A \quad \rightarrow \quad G_{\mathrm{allowed}}$

$SU(2), \quad N_f = 4: \qquad N_f N_c = 8$

$SU(3), \quad N_f = 8: \qquad N_f N_c = 24$

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Both

- satisfy the anomaly condition that permits a symmetric gapped phase
- asymptotically free
- lattice studies suggest they lie near/above the lower edge of the conformal window
- with staggered fermions show the same unusual phase structure

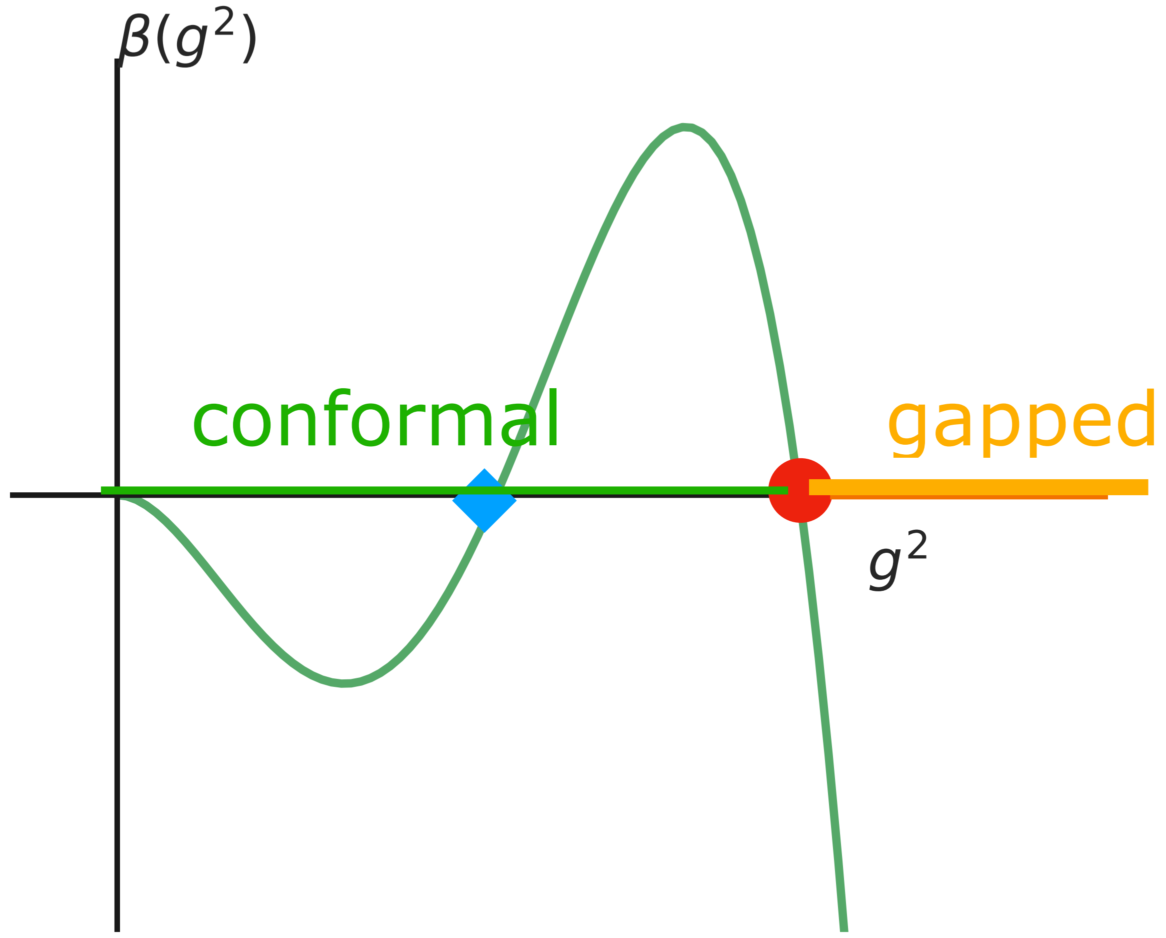
weak coupling ● strong coupling g^2

conformal, gapless $\leftrightarrow$ confining, fully gapped, no Goldstones

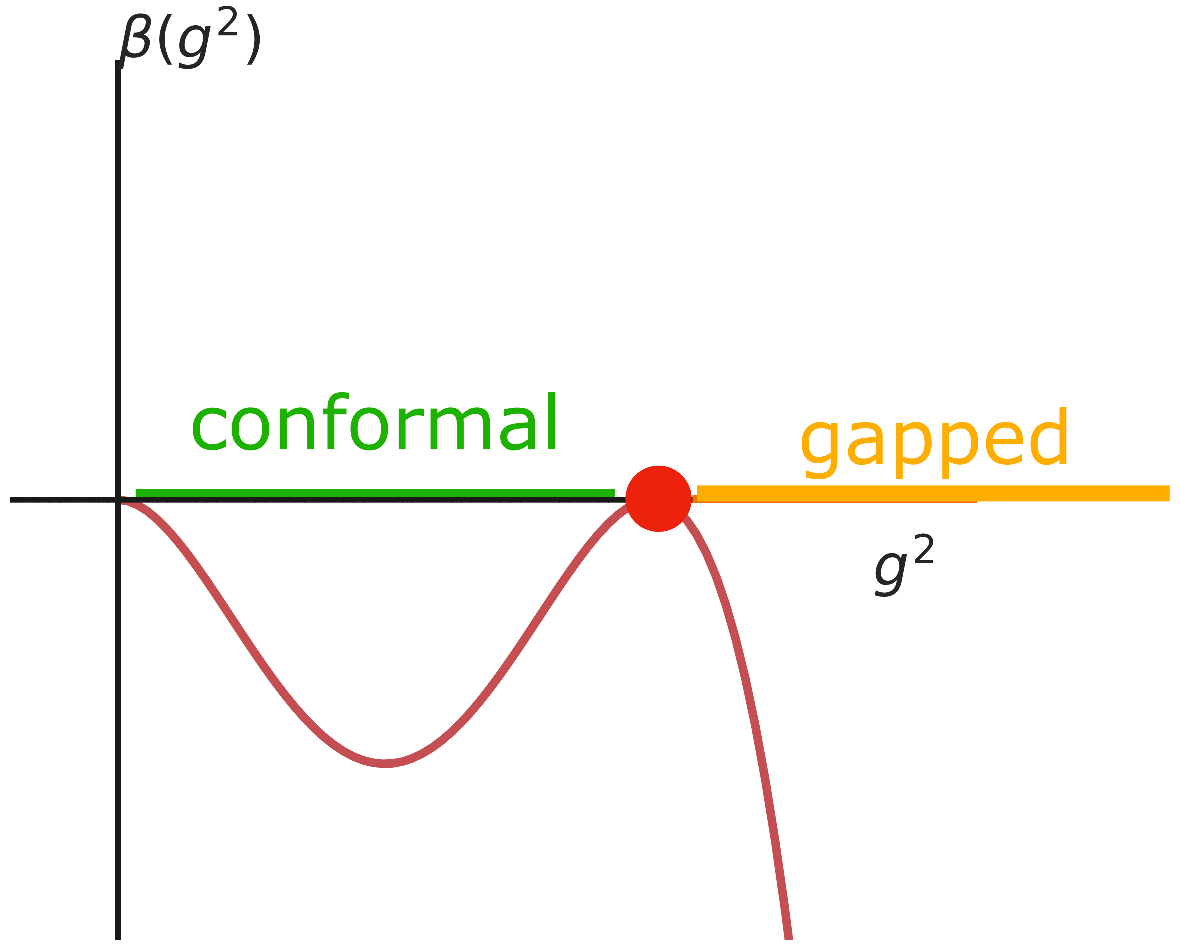
What kind of transition separates the phases?

If it is a continuous phase transition, continuum QFT can be defined

<u>Separate UV and IR fixed points</u>



<u>Merged UV-IR fixed point</u>



"Conformality Lost"
Kaplan et al PRD80,125005 (2009)
L. Vecchi PRD82, 045013 (2010)
Gorbenko et al JHEP10, 108 (2018)

Conformal phase: symmetric and gapless
Strong coupling phase: confining and fully gapped

Conventional continuous transition:

- $\xi \propto |g^2 - g_{\mathrm{UV}}^2|^{-\nu}$

BKT-like/infinite order transition

- $\xi \propto \exp\left(\frac{\zeta}{|g^2 - g_*^2|}\right)$

$\boxed{\text{Which one do the lattice models realize?}}$

Why staggered fermions?

Free staggered action

$$S_{lat} = \sum_{n,\mu} \eta_\mu(n)\,\bar{\chi}(n)\,\Delta_\mu \chi(n) + m \sum_n \bar{\chi}(n)\chi(n), \qquad \eta_\mu(n) = (-1)^{\sum_{k=1}^{\mu-1} n_k}$$

breaks continuum $SU(4)_V \times SU(4)_A \times U(1)_V \times U(1)_A \;\longrightarrow\; U(1)_V \times U(1)_\epsilon \times \Gamma_4$

$U(1)_\epsilon \not\equiv U(1)_A$ but $\mathrm{Spin} - Z_4 \in \Gamma_4$ (Γ_4 is shift symmetry)

Symanzik effective continuum action

Lee,Sharpe PRD60, 114503 (1999)

$$S_{\rm eff} = S_{\rm cont} + a^2 \sum c_i O_i^{(6)} + \ldots$$

Dimension-6 operators include taste breaking 4-fermion interactions, e.g.

$$\mathcal{L}_{eff} = \bar{\psi}(x) D_\mu \gamma_\mu \psi(x) + u\left[(\bar{\psi}(x)\psi(x))^2 - (\bar{\psi}(x)\gamma_5\xi_5\psi(x))^2\right]$$

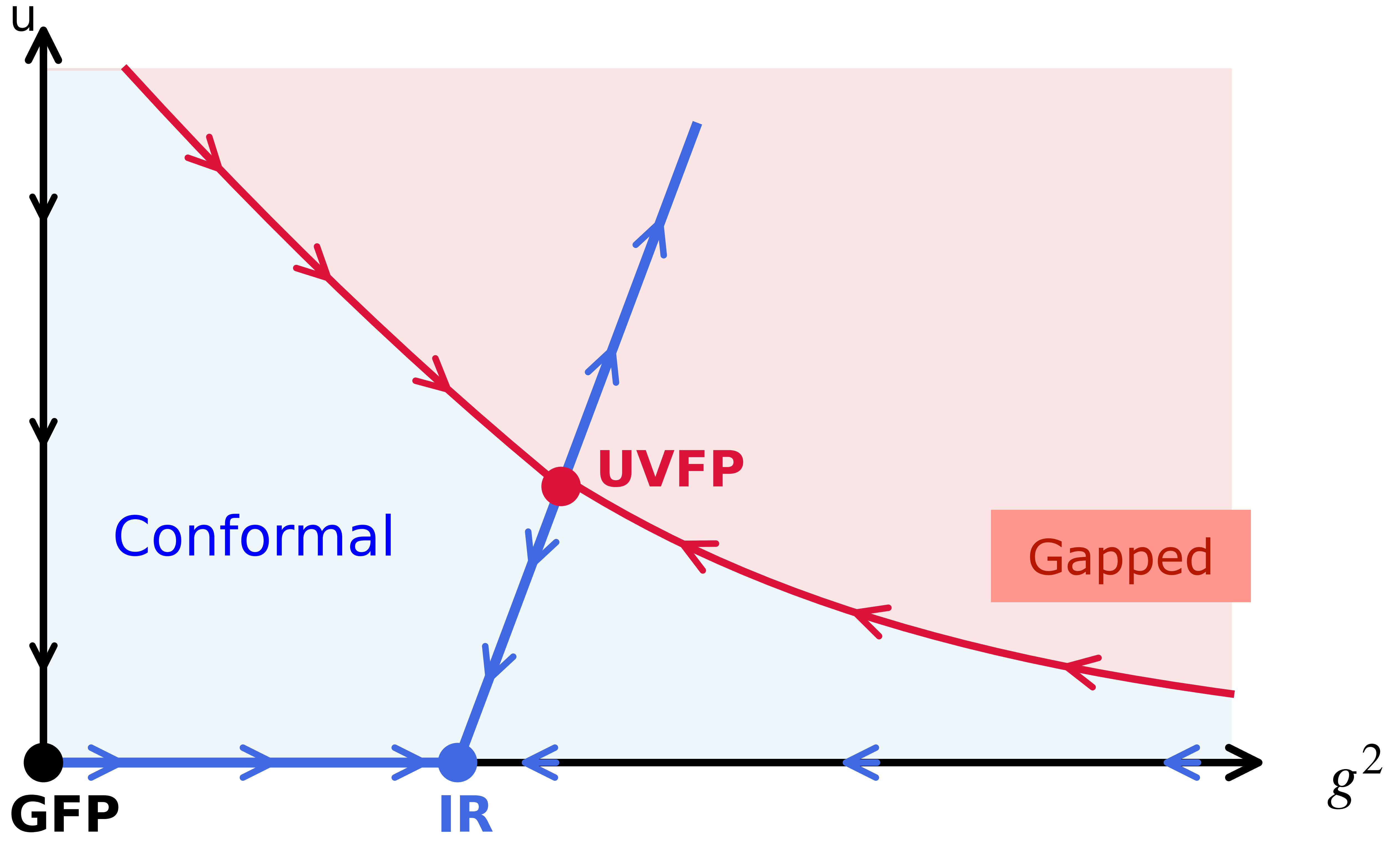
u is irrelevant at GFP, but it can become relevant at a strongly coupled FP

Possible RG structure I: separate IR and UV fixed points

AH, Cenke Xu, arXiv:2604.02424
T. Yamaoka, private comm.
based on Swinger-Dyson

(g^2, u) parameter space, within the conformal window

g^2 : gauge coupling, u : symmetry-reducing four-fermion interactions



Fixed points:
- ● GFP
- ● IR
- ● UV critical point

red line: phase transition

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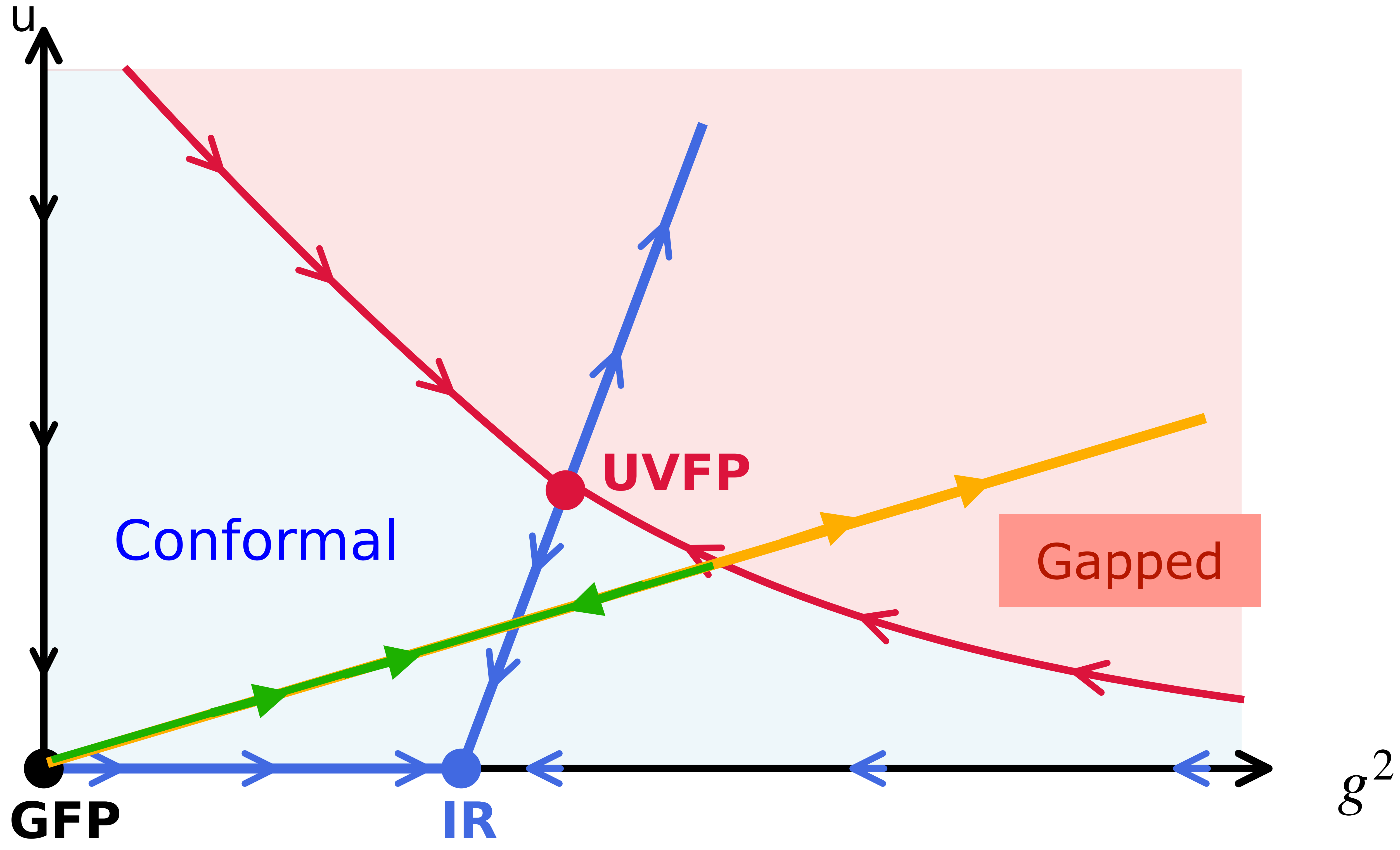

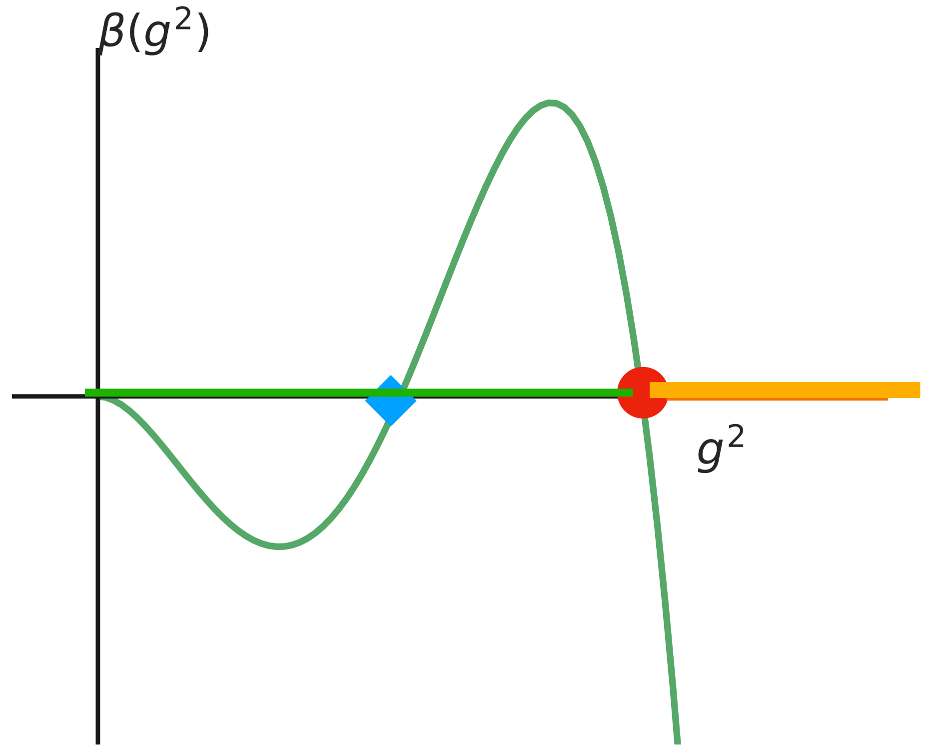

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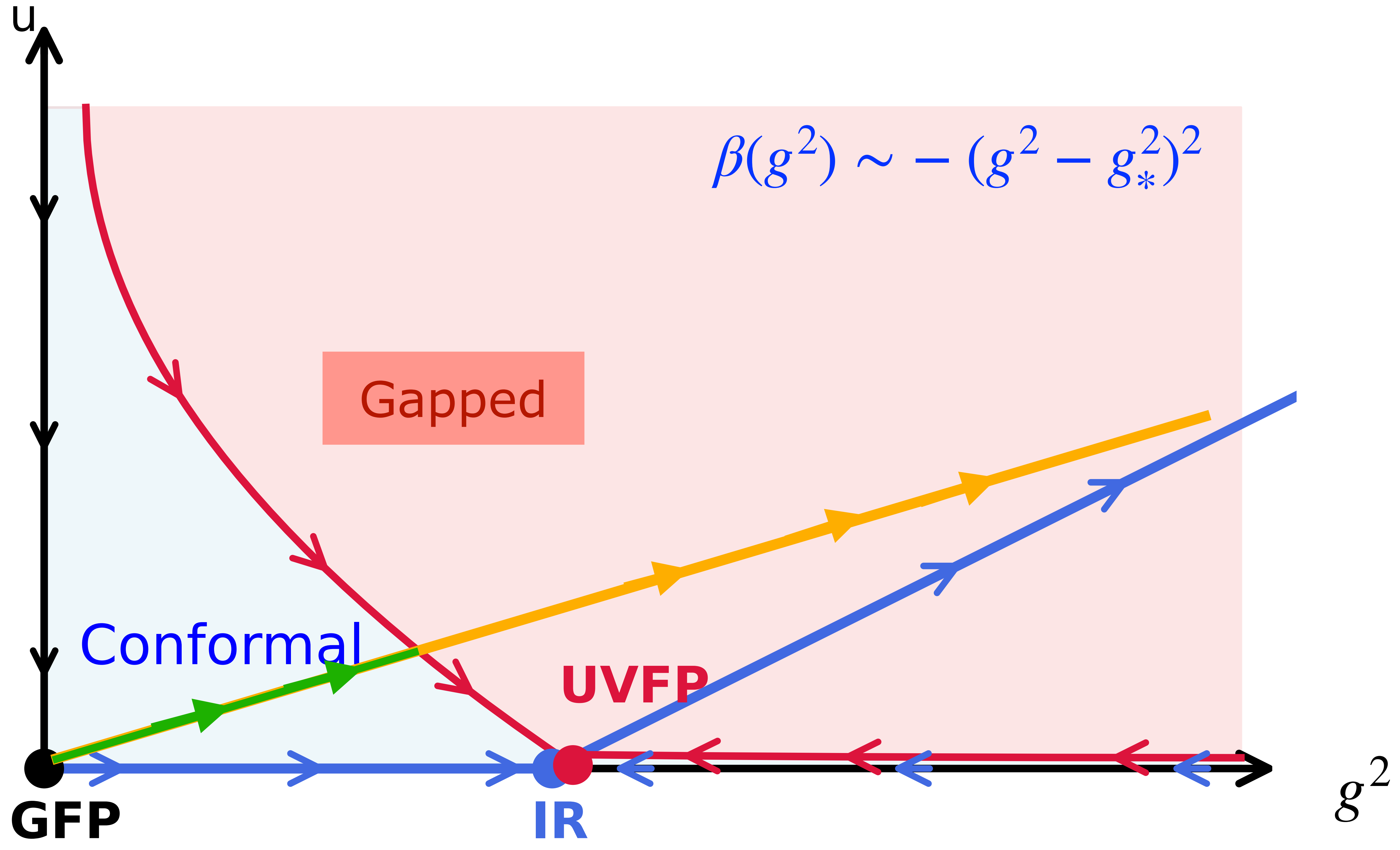

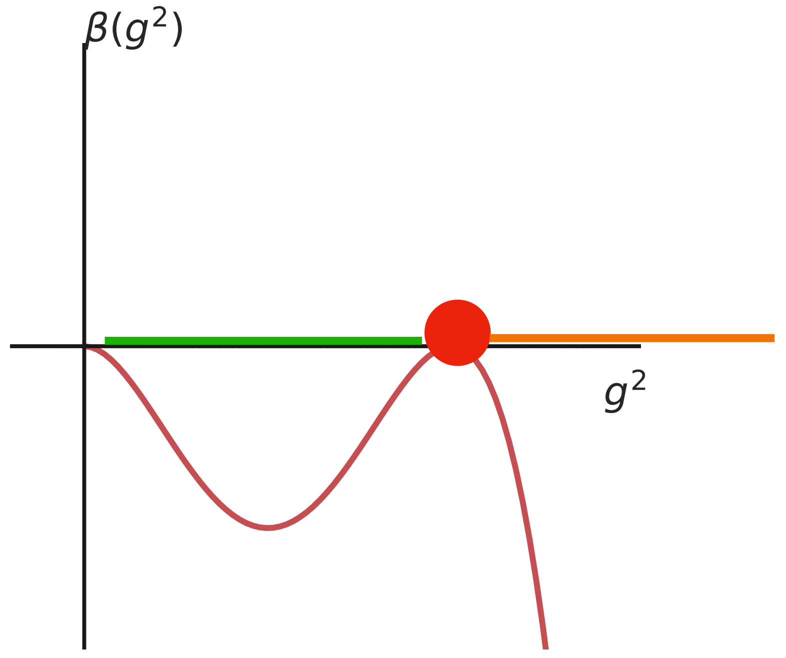

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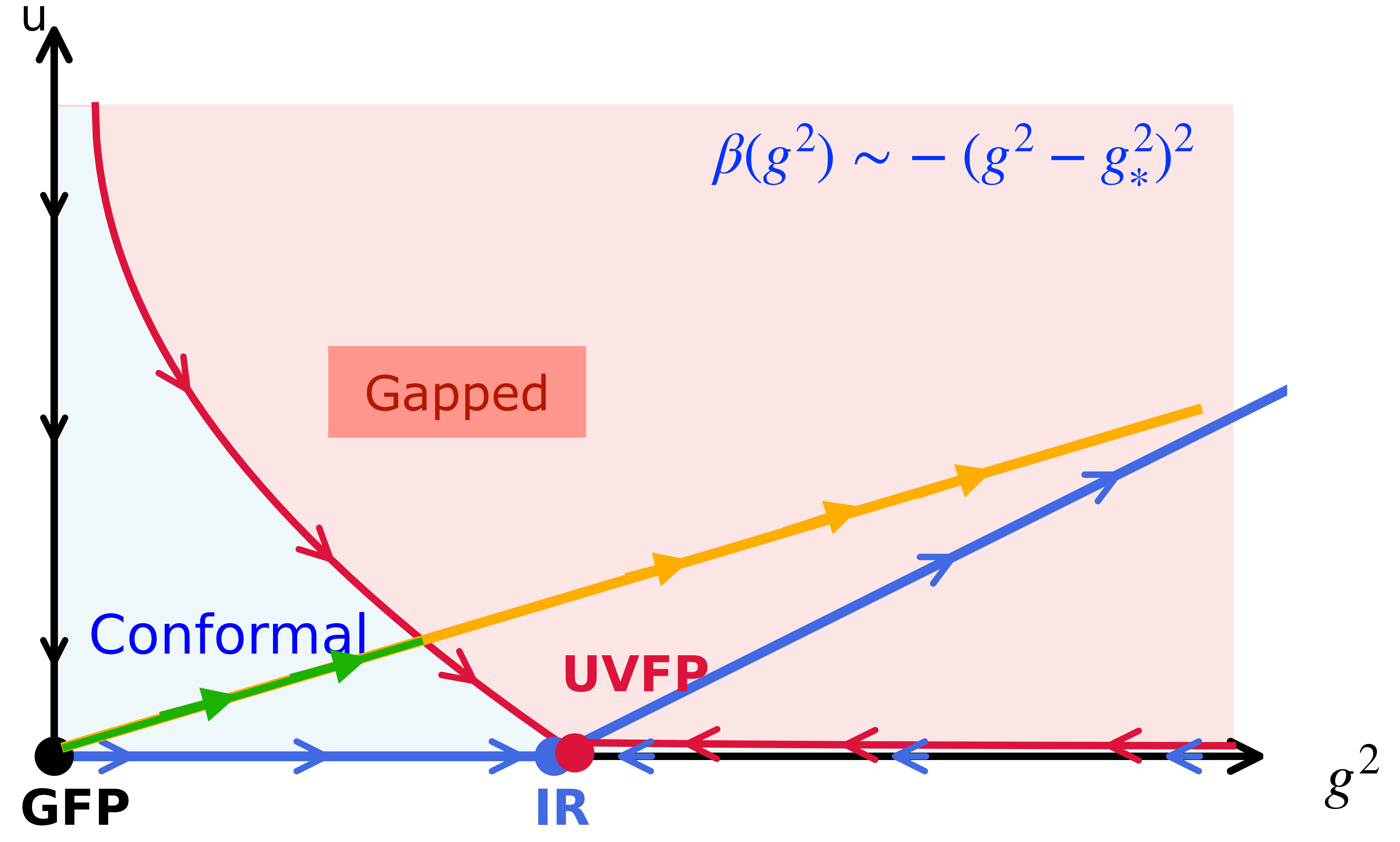

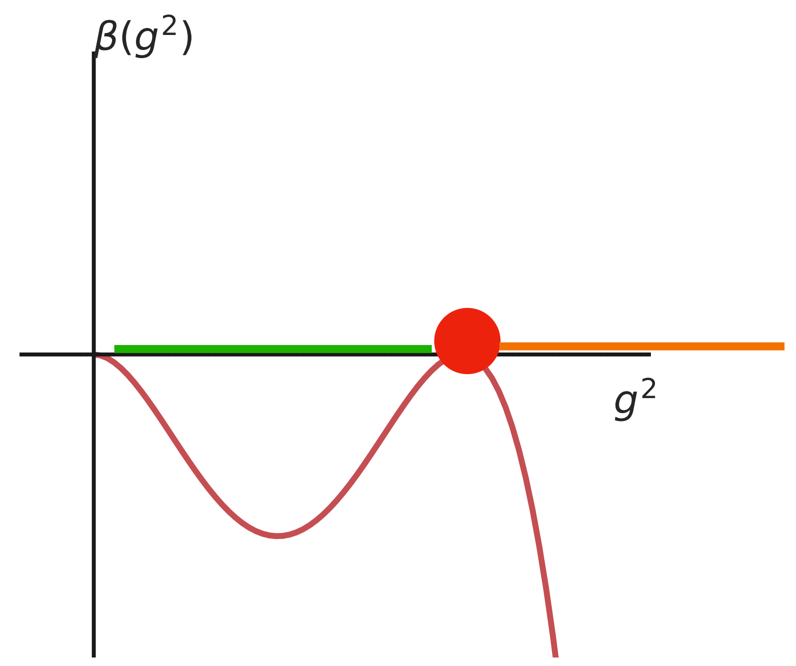

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When UV and IR merge, the UVFP inherits the symmetries of the IRFP

Emergent taste symmetry distinguishes the two scenarios

Staggered fermions break $SU(N_f)_L \times SU(N_f)_R$, meson tastes are split

$\Longrightarrow R \equiv \dfrac{M_{\text{PS}}}{M_{\text{PS2}}} < 1$ in general; $R \to 1$ at the GFP and IRFP in $L \to \infty$ limit

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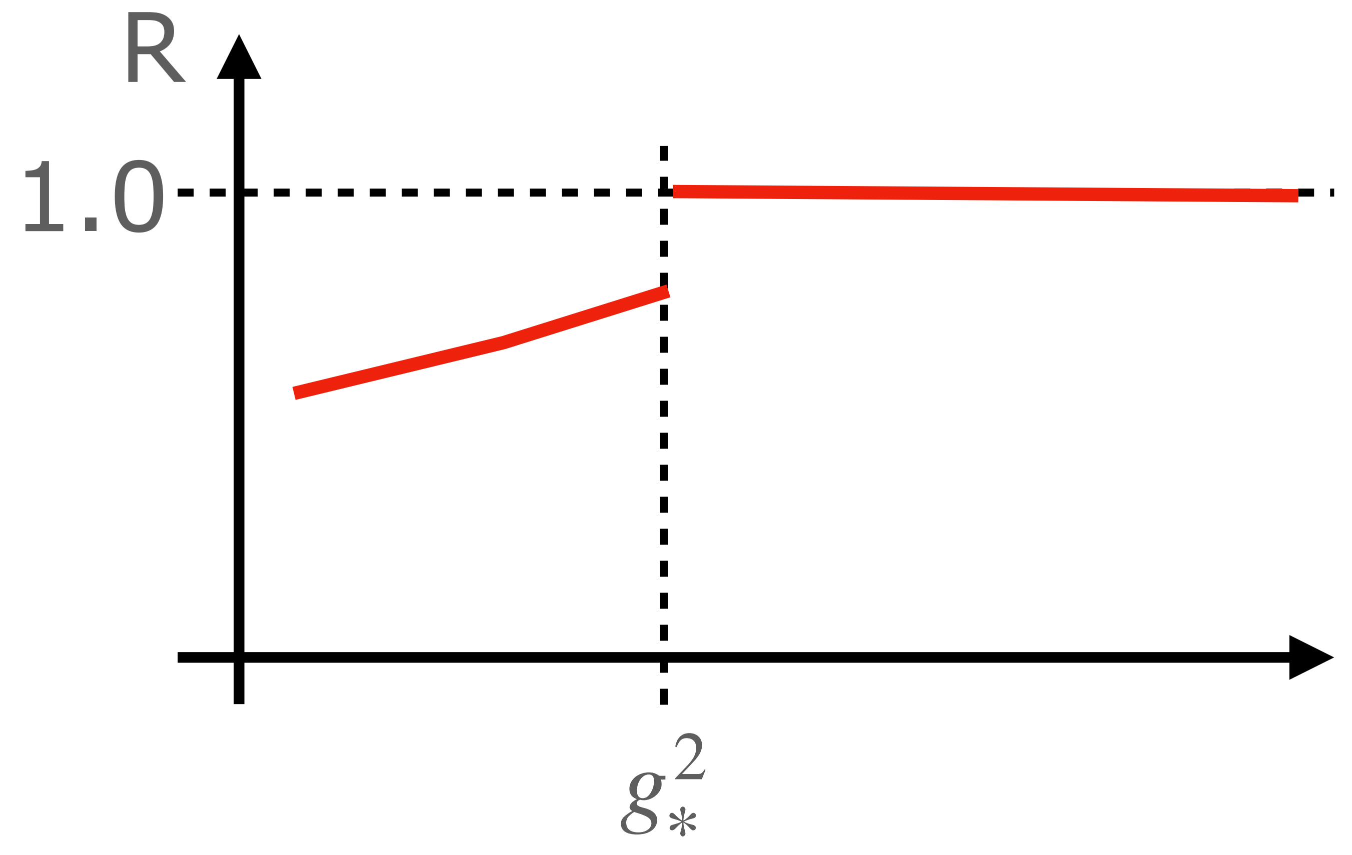
<u>Separate UV and IR fixed points:</u>

- UVFP can have lower symmetry
- strong coupling gapped phase exhibits taste breaking
- $R_c \neq 1$ is allowed
- R **can jump at the transition**



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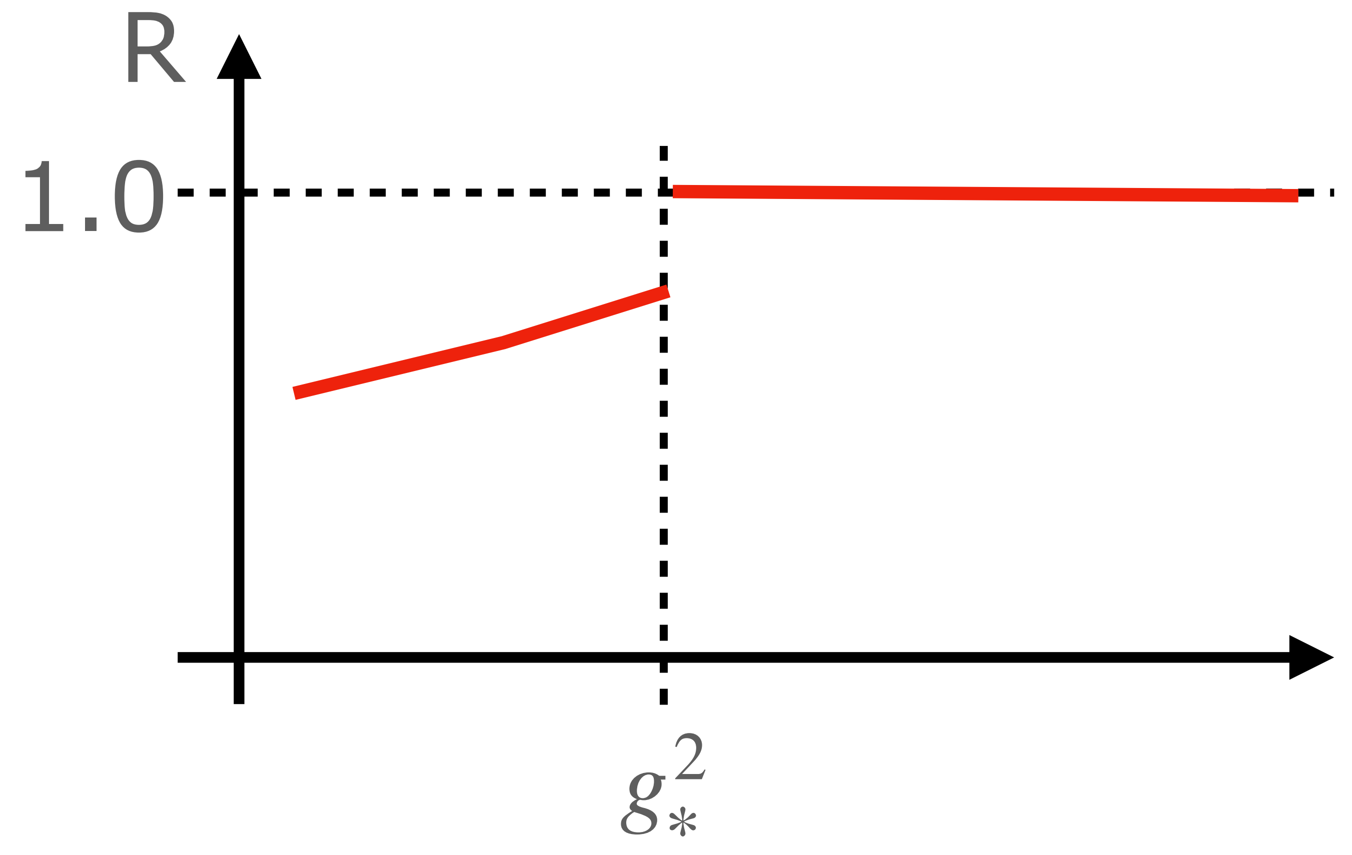
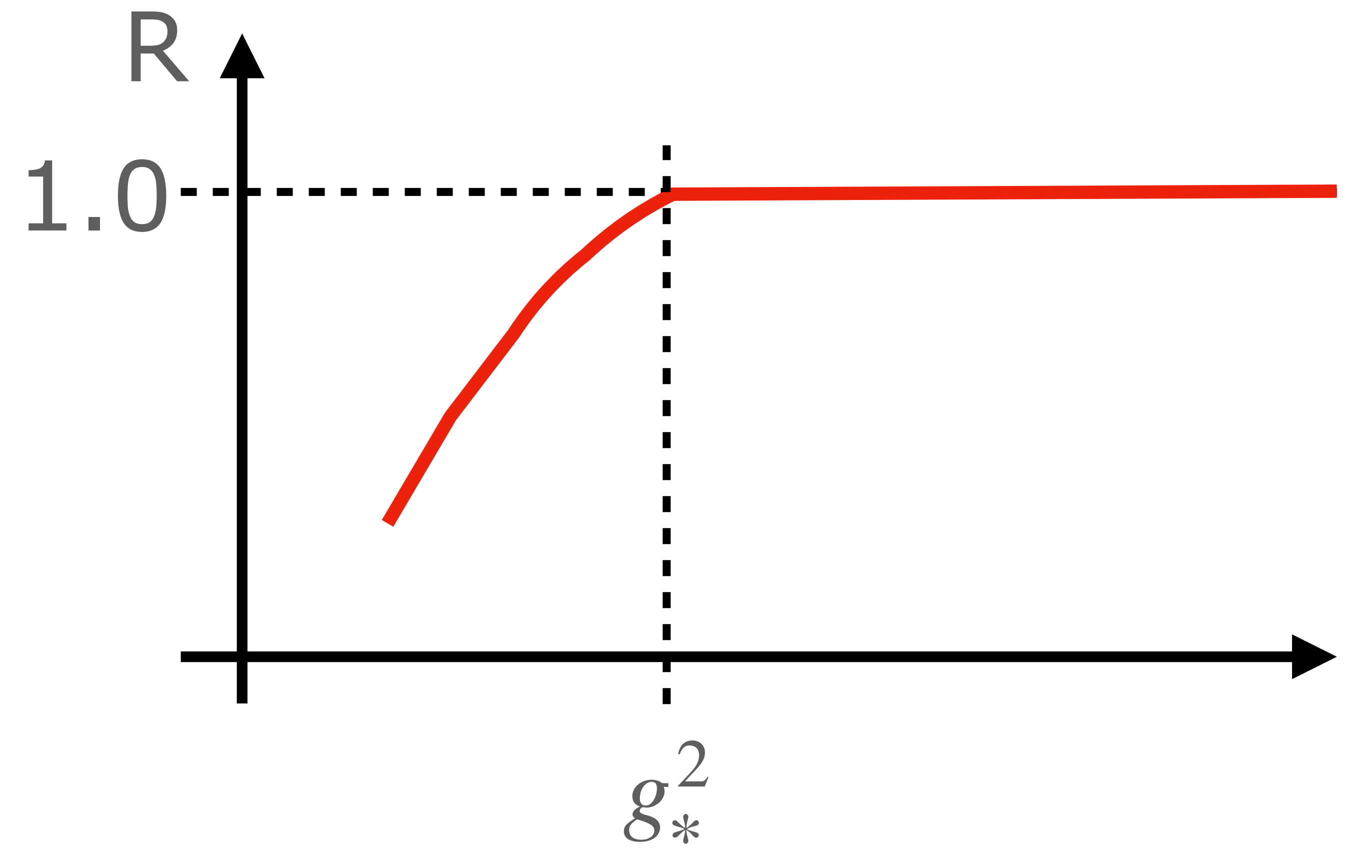


<u>Merged UV-IR fixed point:</u>

- IRFP = UVFP
- merged FP inherits full $SU(4)_L \times SU(4)_R$ symmetry
- $R \to 1$ at the critical point
- R **is continuous across the transition**



Numerical results: phase diagram

AH, *PRD* 106 (2022) 1, 014513
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The two systems

- SU(3) gauge + $N_f = 8$; SU(2) gauge + $N_f = 4$

look very similar:

All simulations are directly in the $m_f = 0$ chiral limit!

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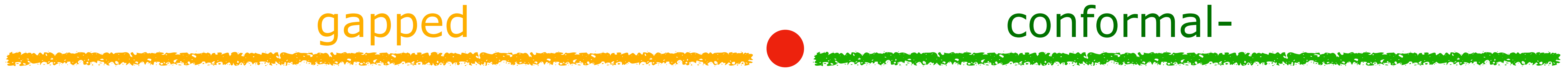

$\beta_b = N_c/g^2$

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- the phase transition is continuous
  $\rightarrow \exists$ continuum limit and RG β function

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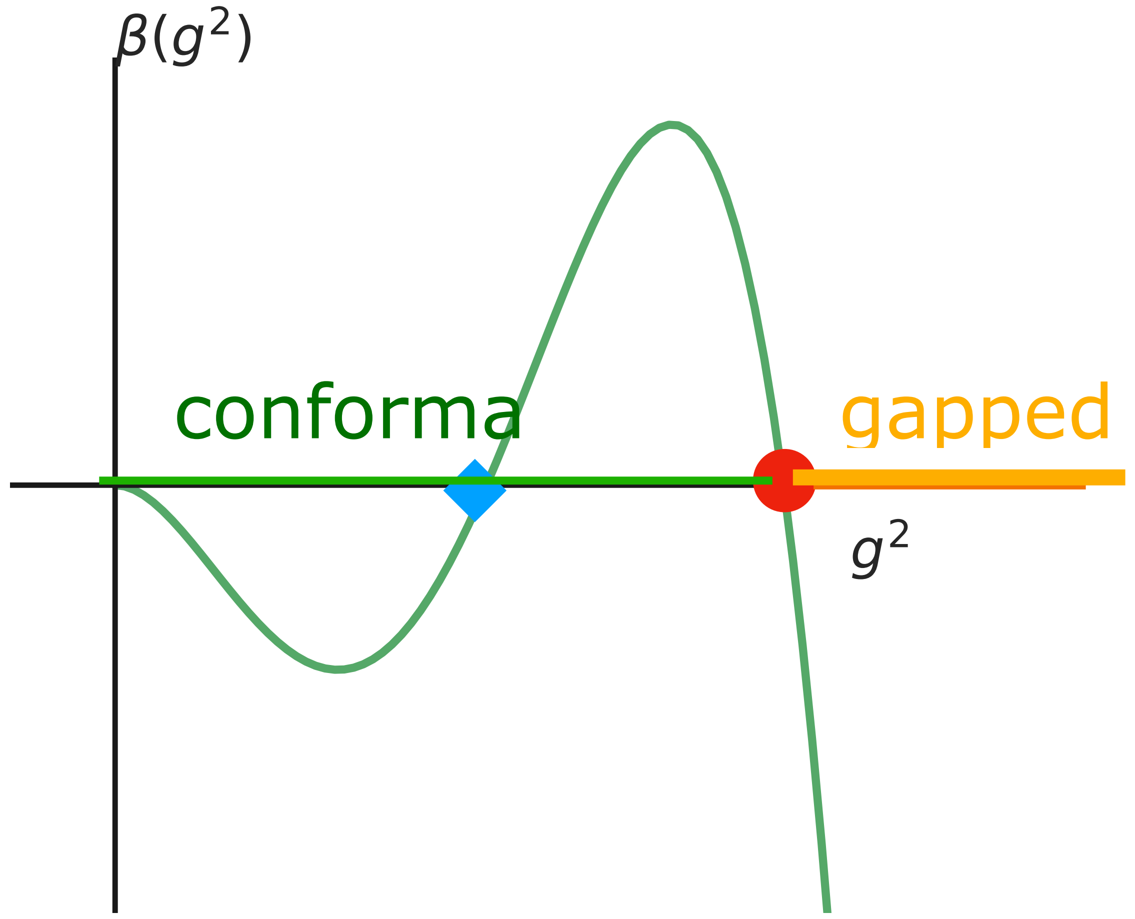

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- it could be “walking”: fixed point merger leads to the opening of the conformal window

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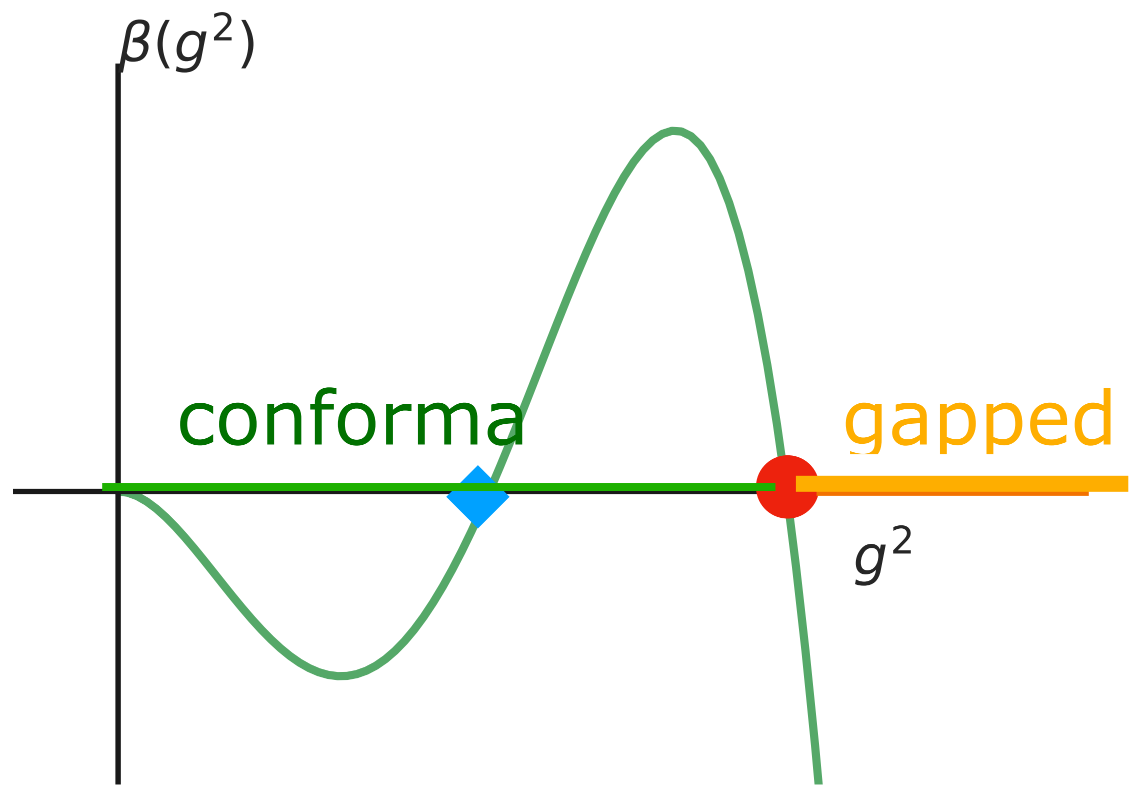
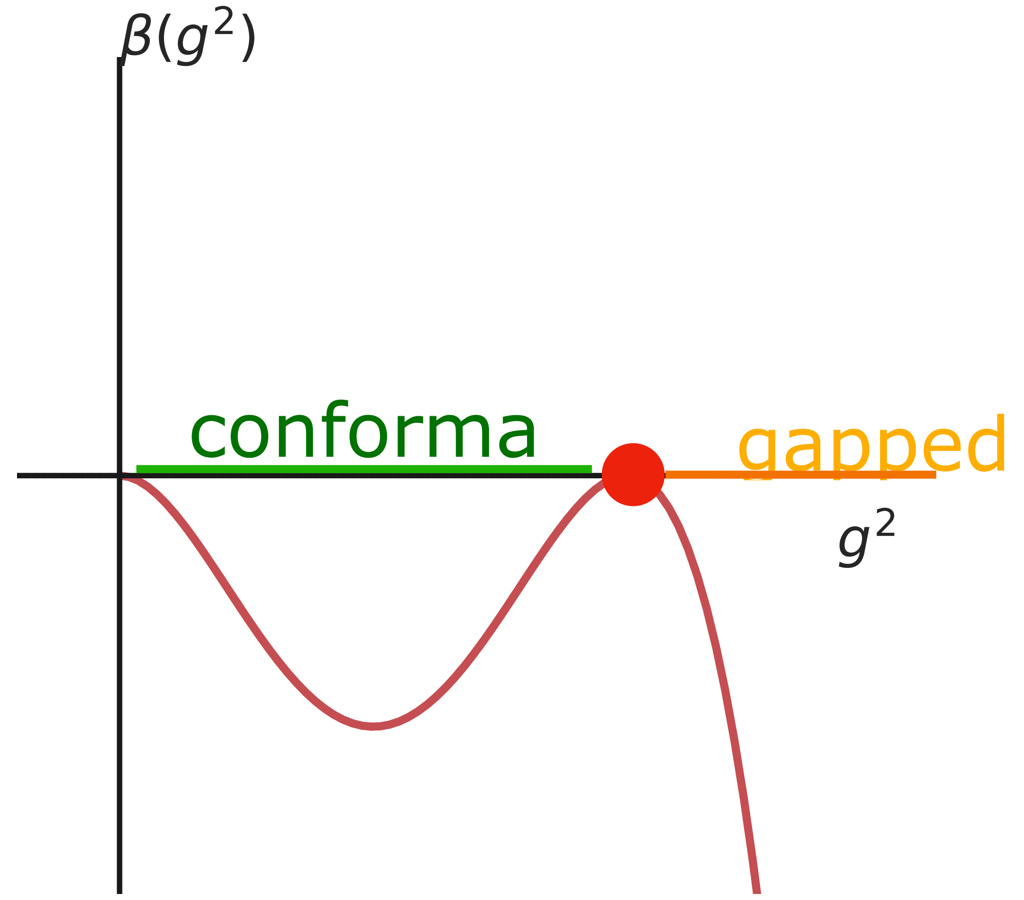
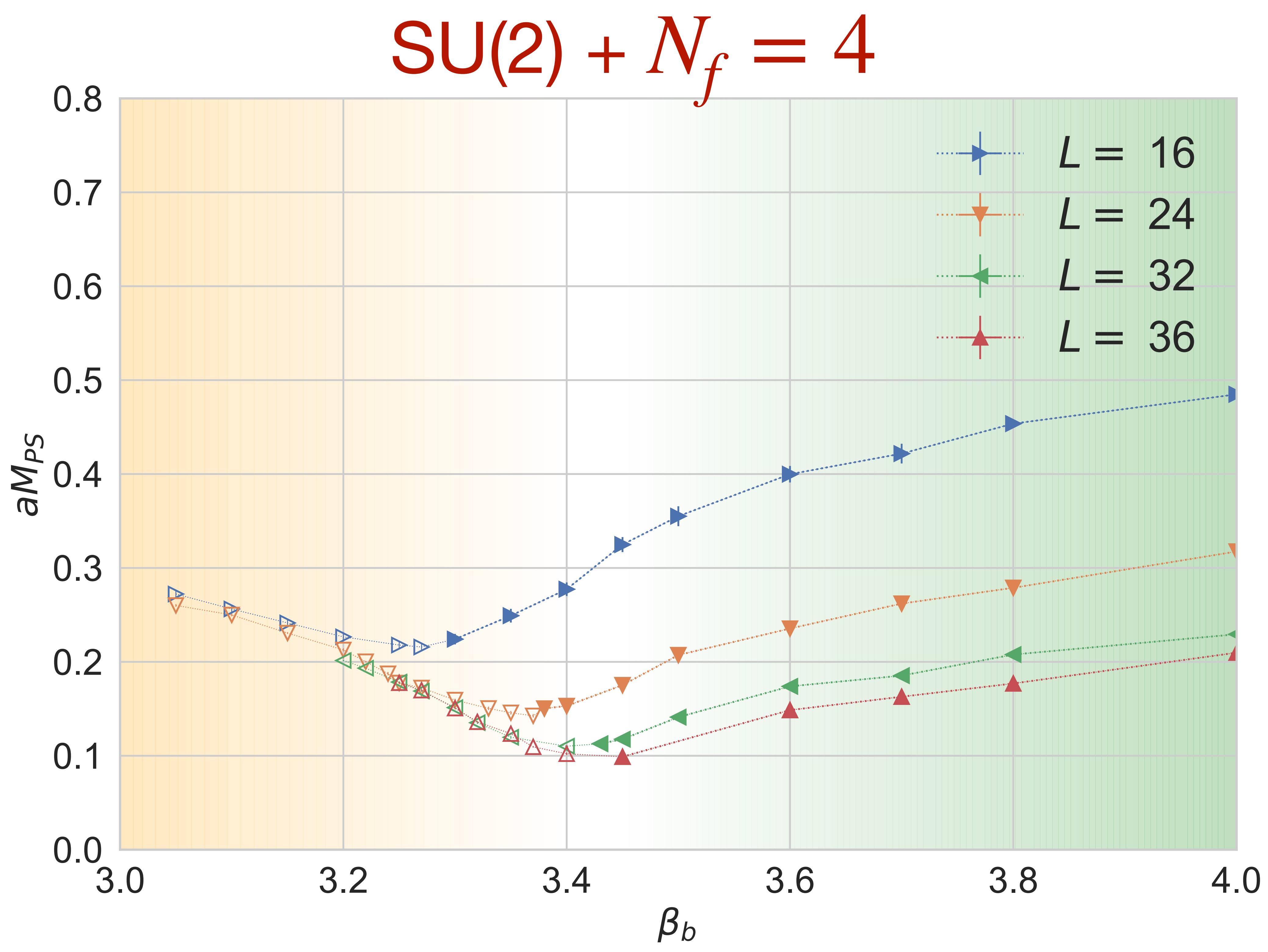
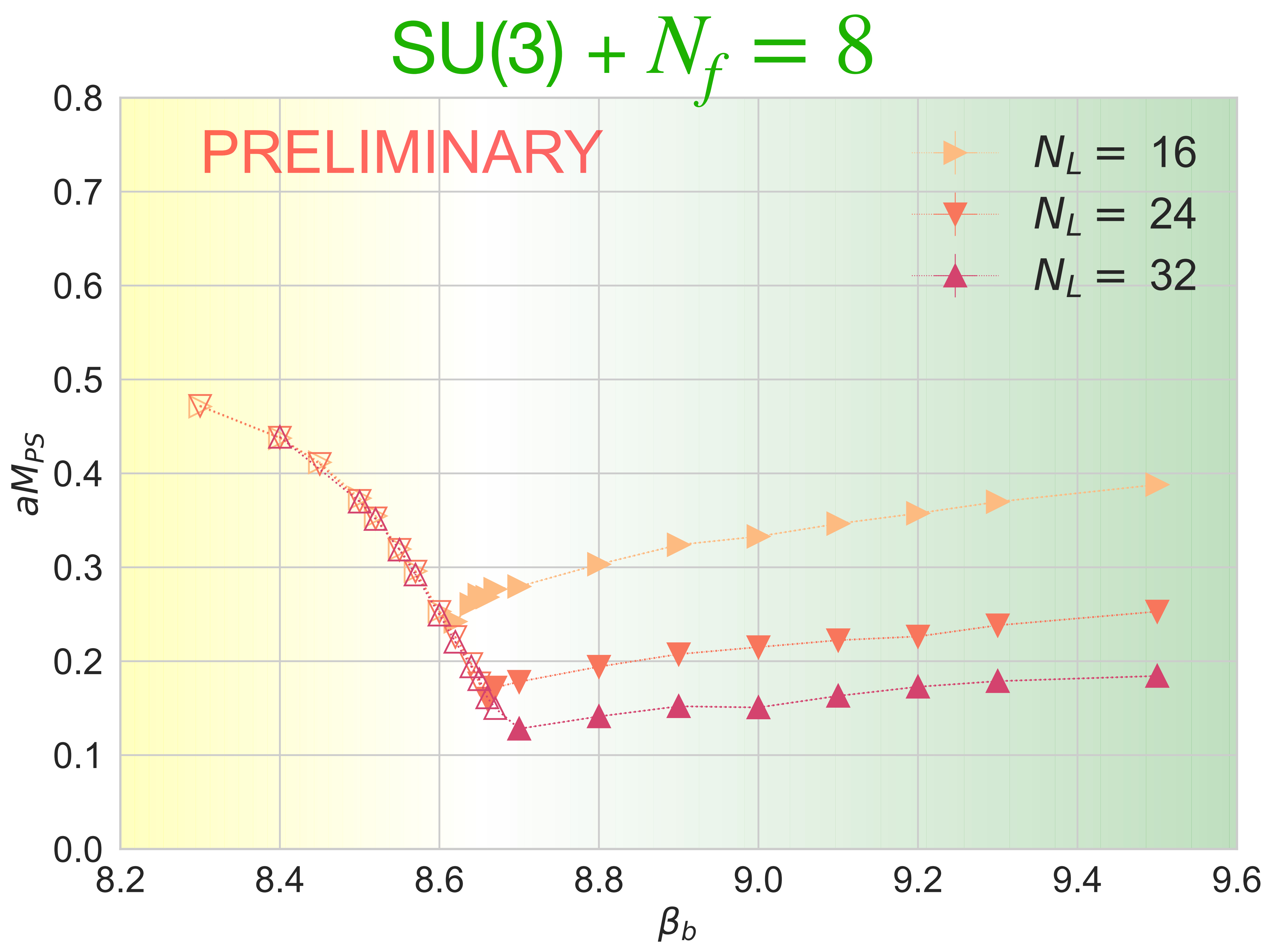




Strong coupling is gapped, weak coupling is gapless

Mass of would-be Goldstone pion (PS)



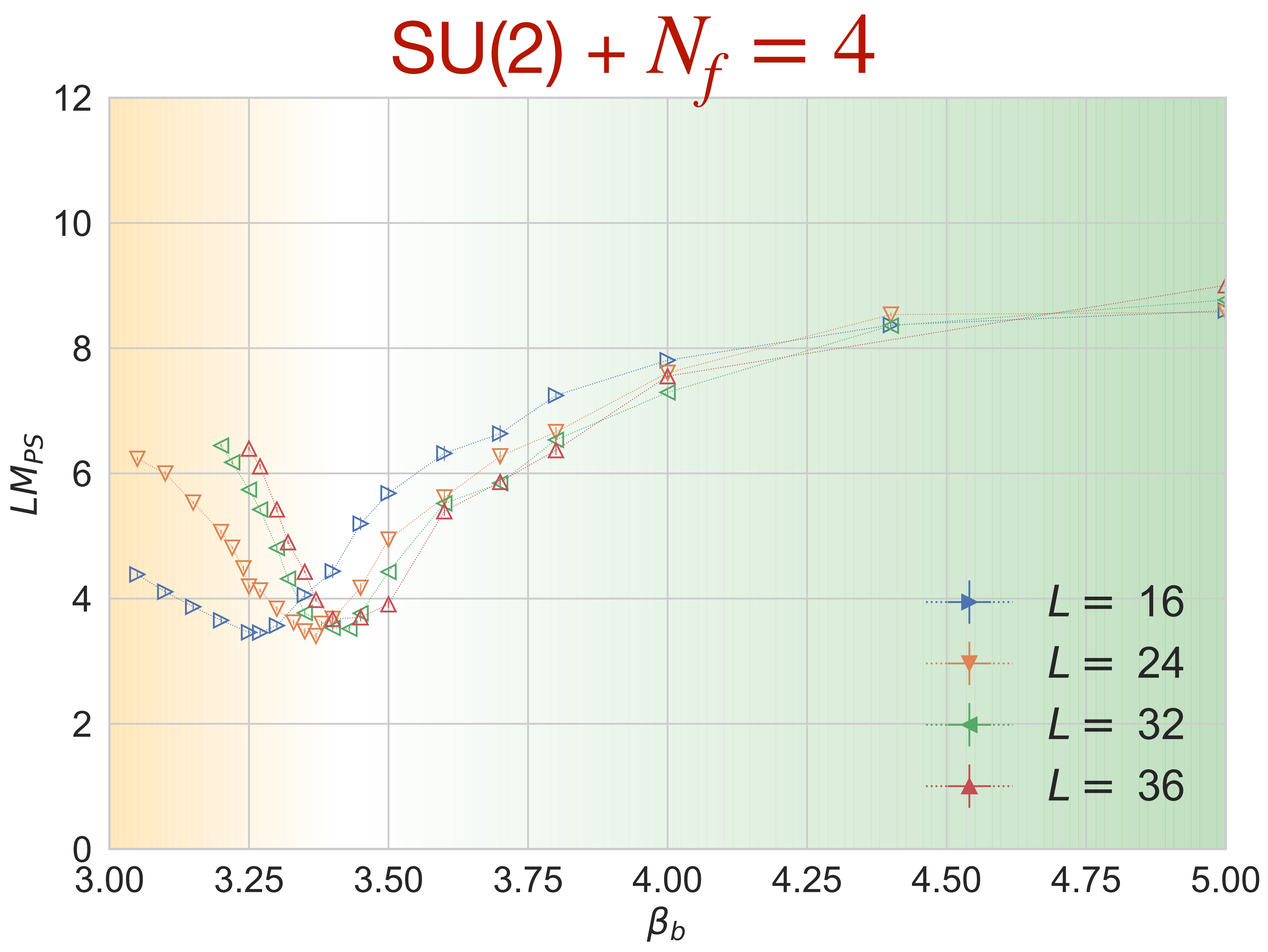


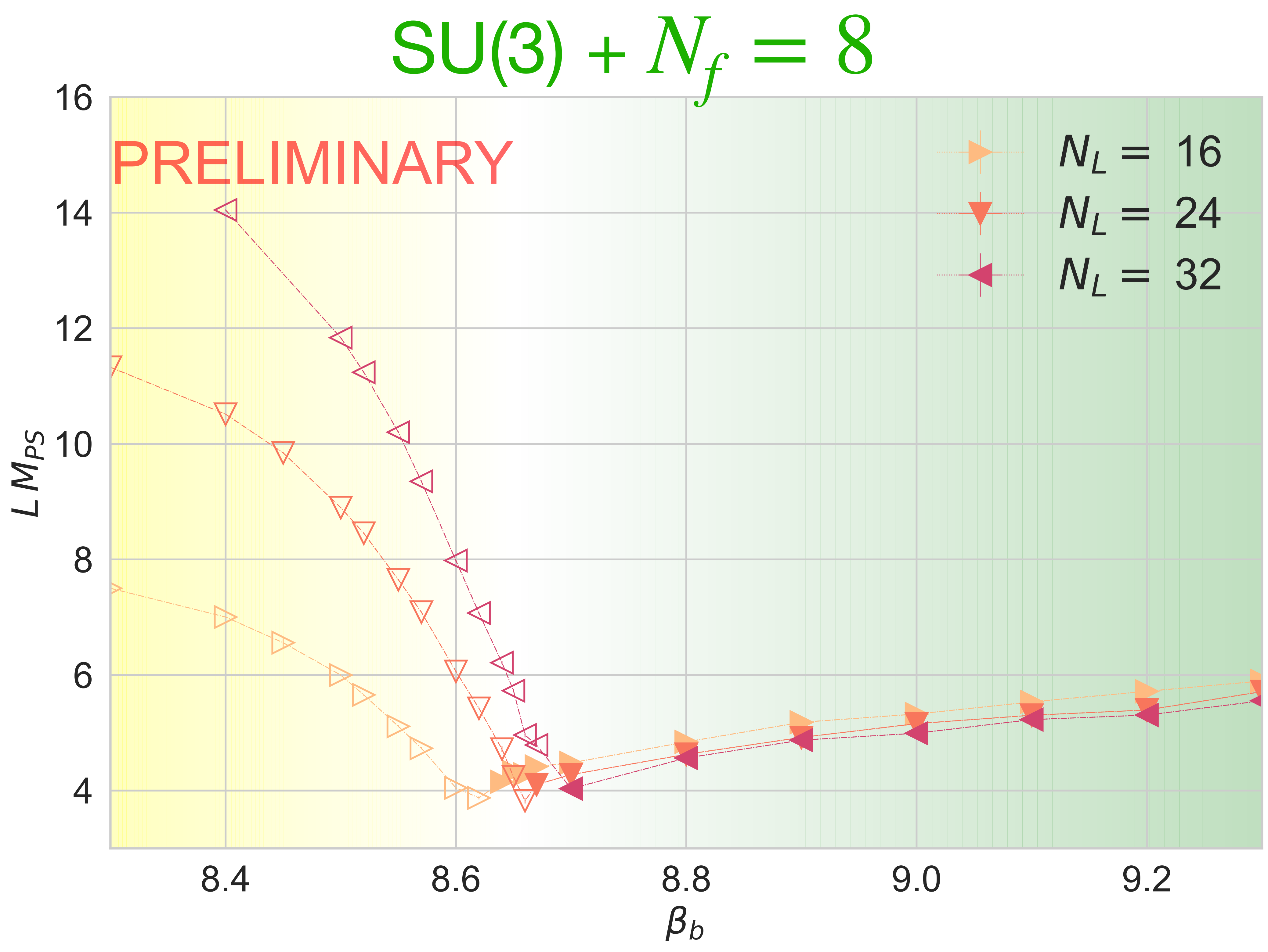
approximately volume independent "gapped" phase

1/L finite size scaling consistent with gapless phase

Weak coupling : $M_{PS}L \approx \text{const}$

Weak coupling: $M_{PS}L \approx$ const
PS channel is consistent with conformal finite-volume scaling



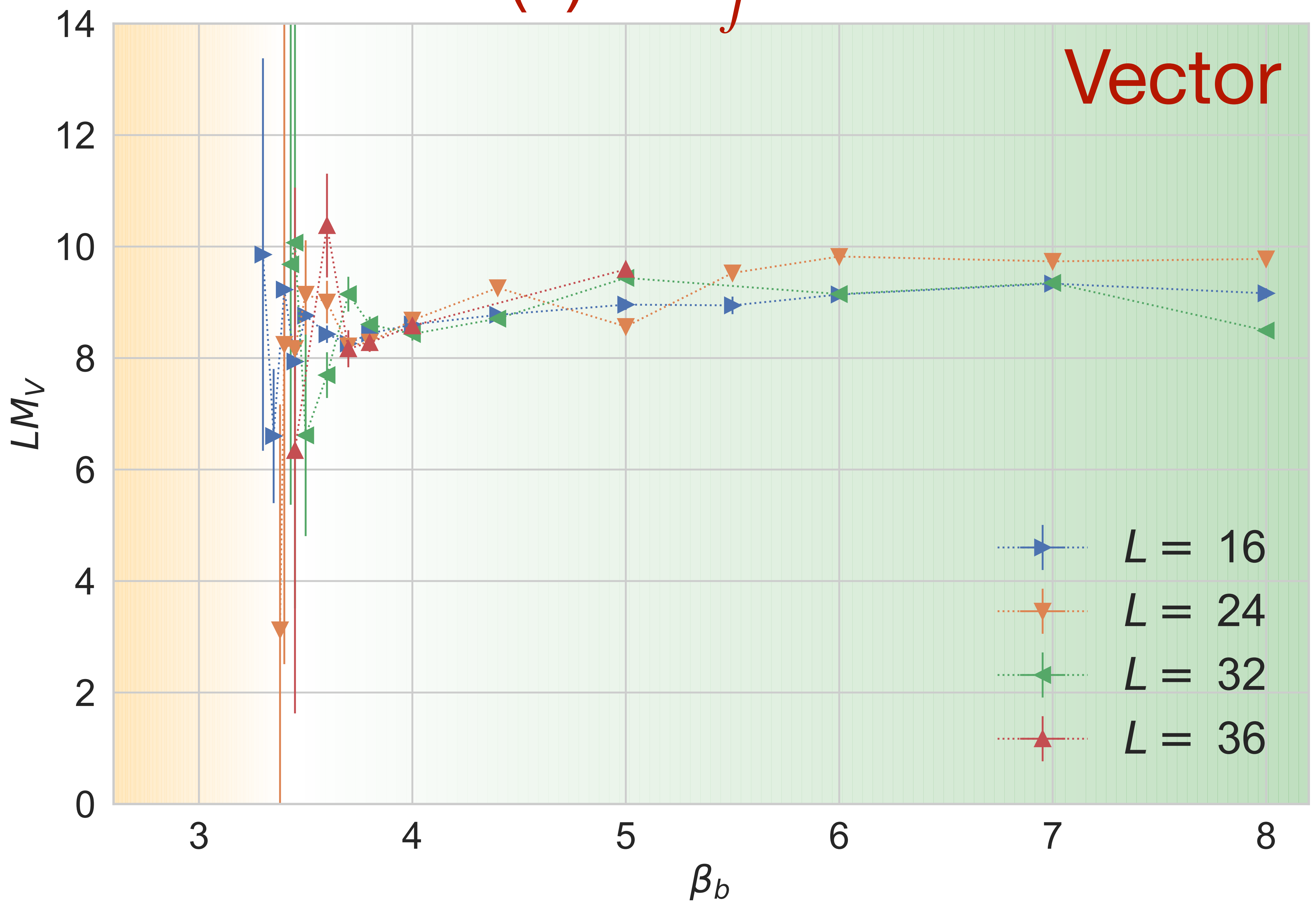


Both systems show $M_{PS}L \approx \text{const}$ in the weak coupling side

Weak coupling : $M_V L \approx \text{const}$

Weak coupling regime shows $M_V L \approx$ const
It is not unique to the PS channel

SU(2) + $N_f = 4$



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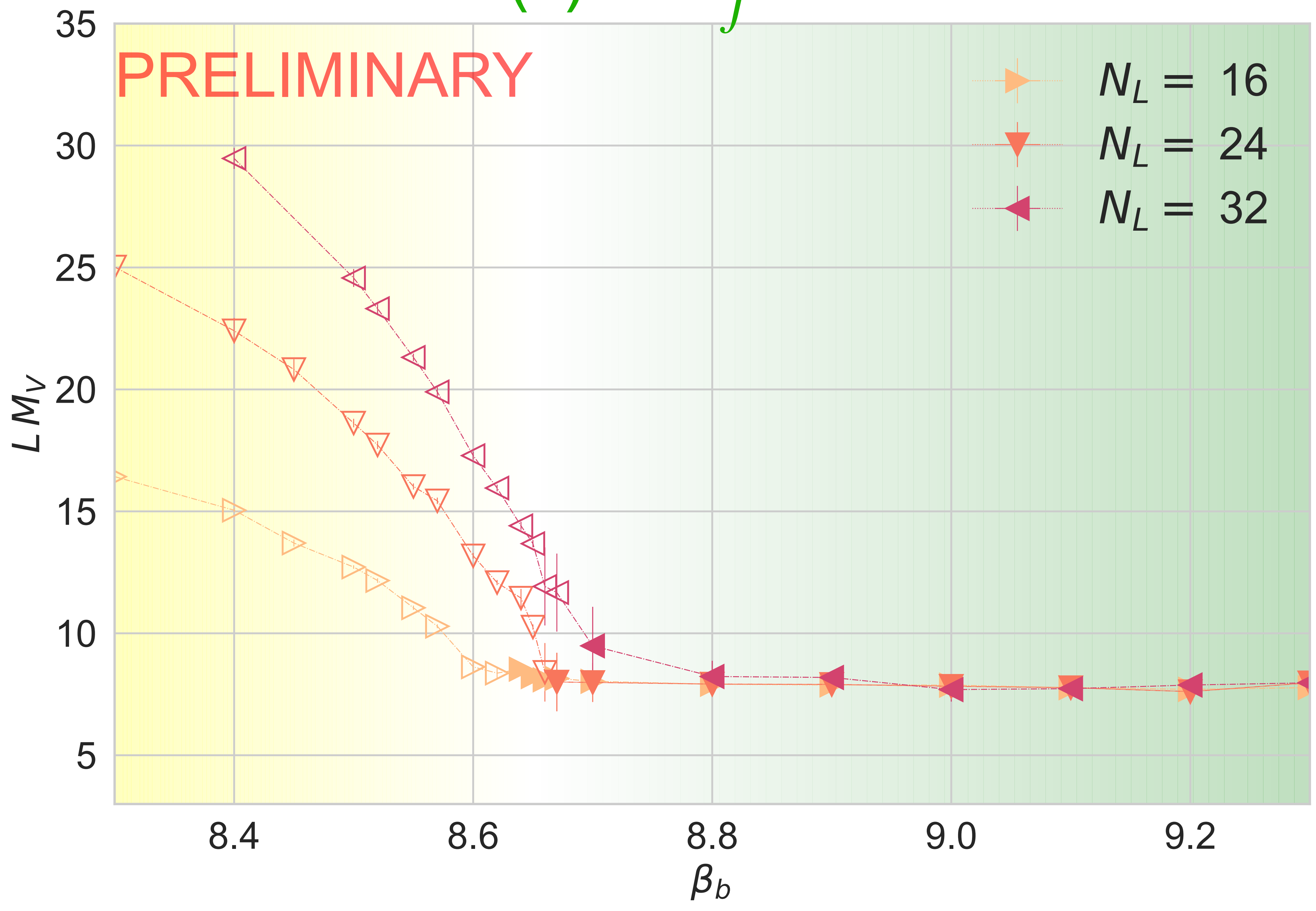
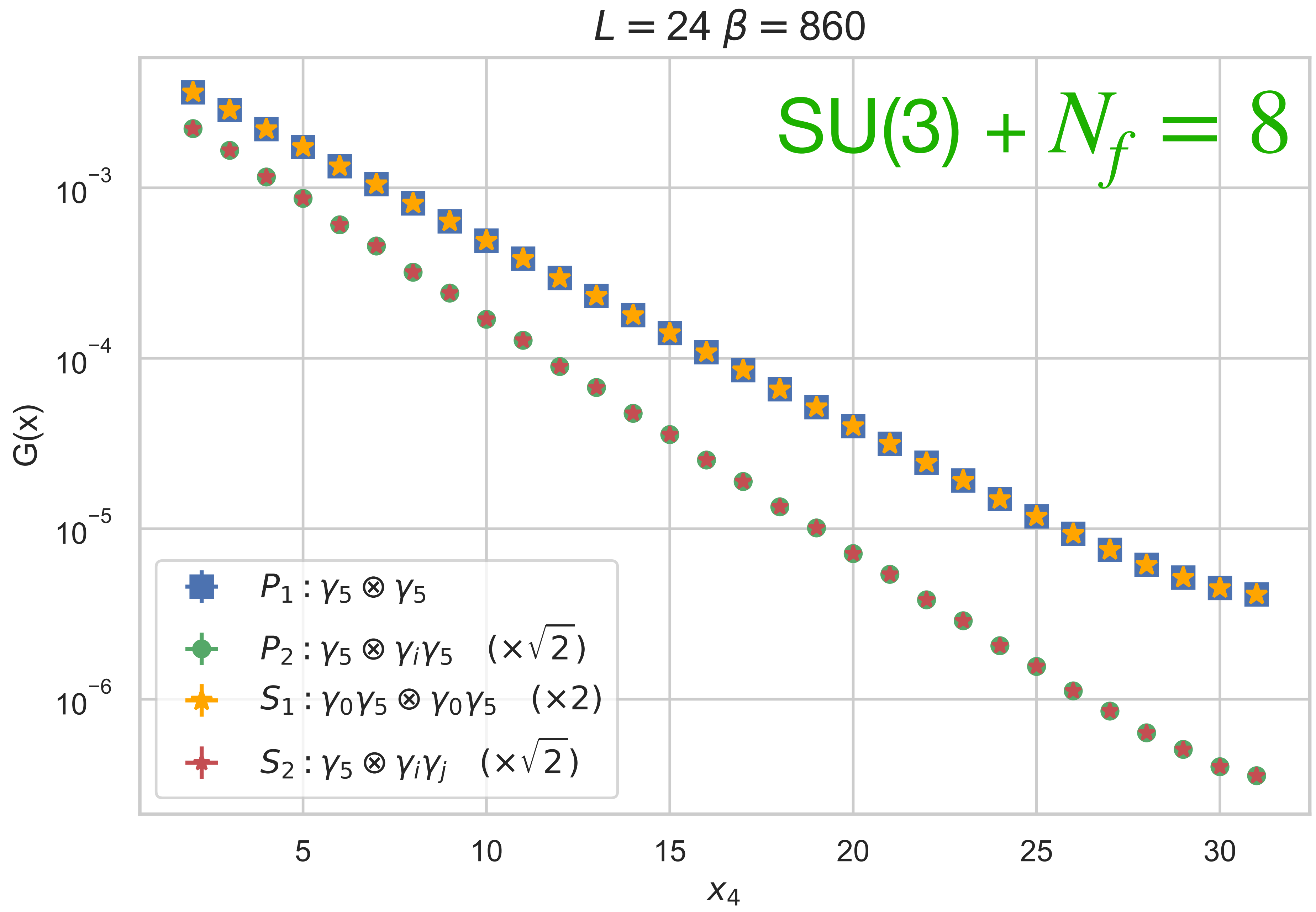
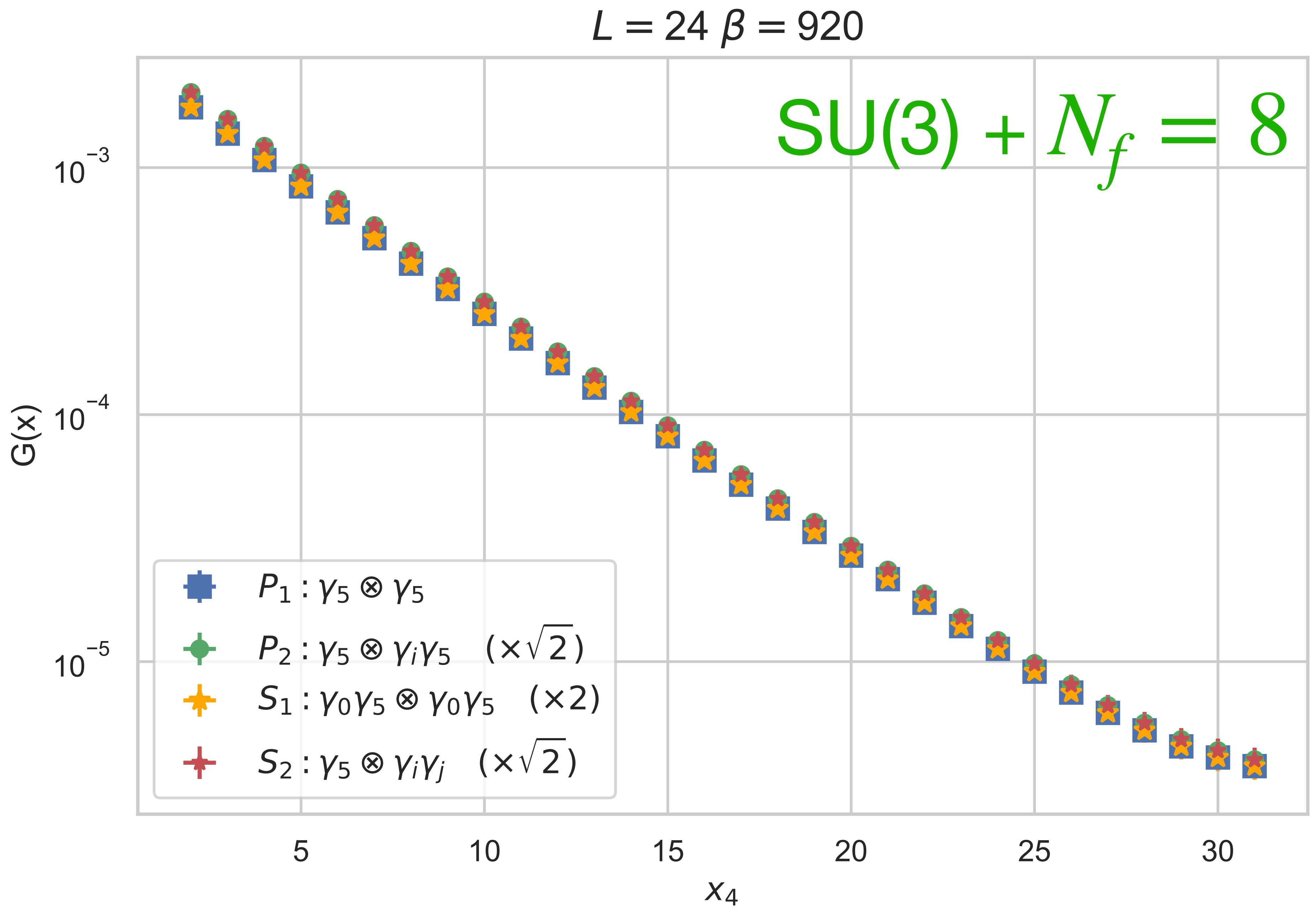


Vector channel confirms conformal hyper scaling

No chiral symmetry breaking, but taste breaking

Parity partner correlators ($P_1 - S_1$ and $P_2 - S_2$) are identical in both phases : chiral symmetry to machine precision!





strong coupling phase
significant taste braking

weak coupling phase
approximate taste restoration

Phase diagram, so far

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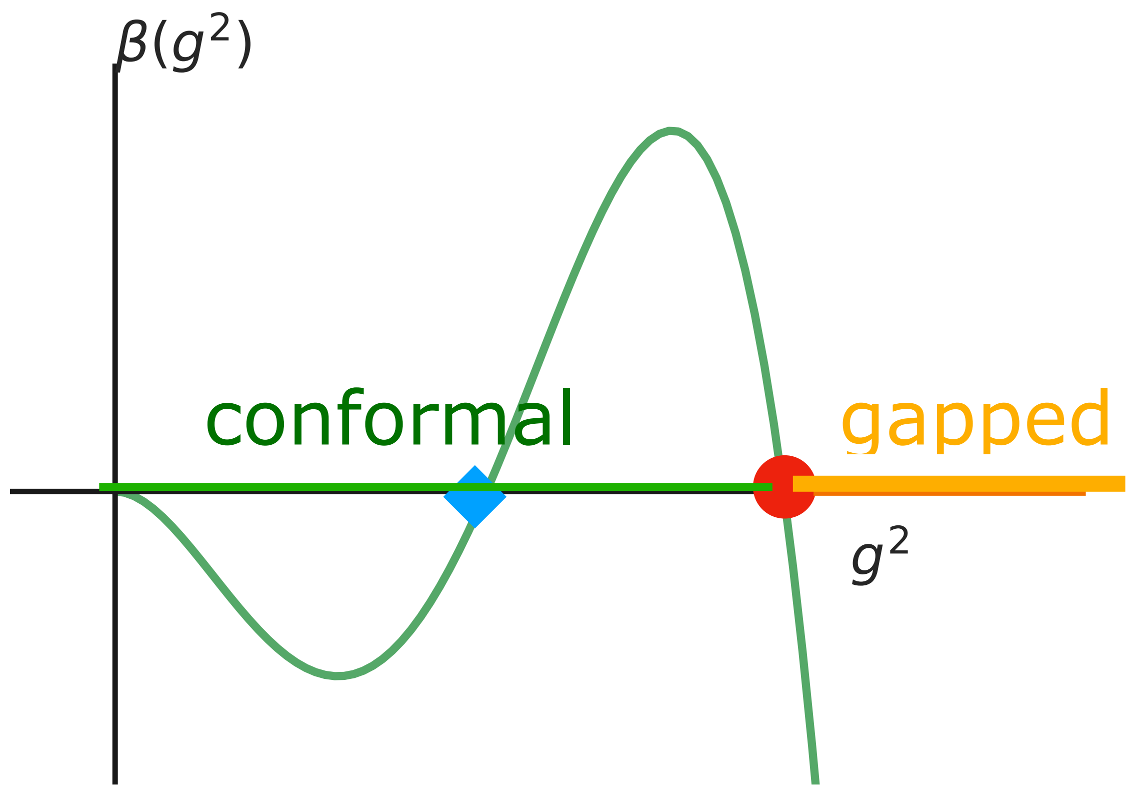
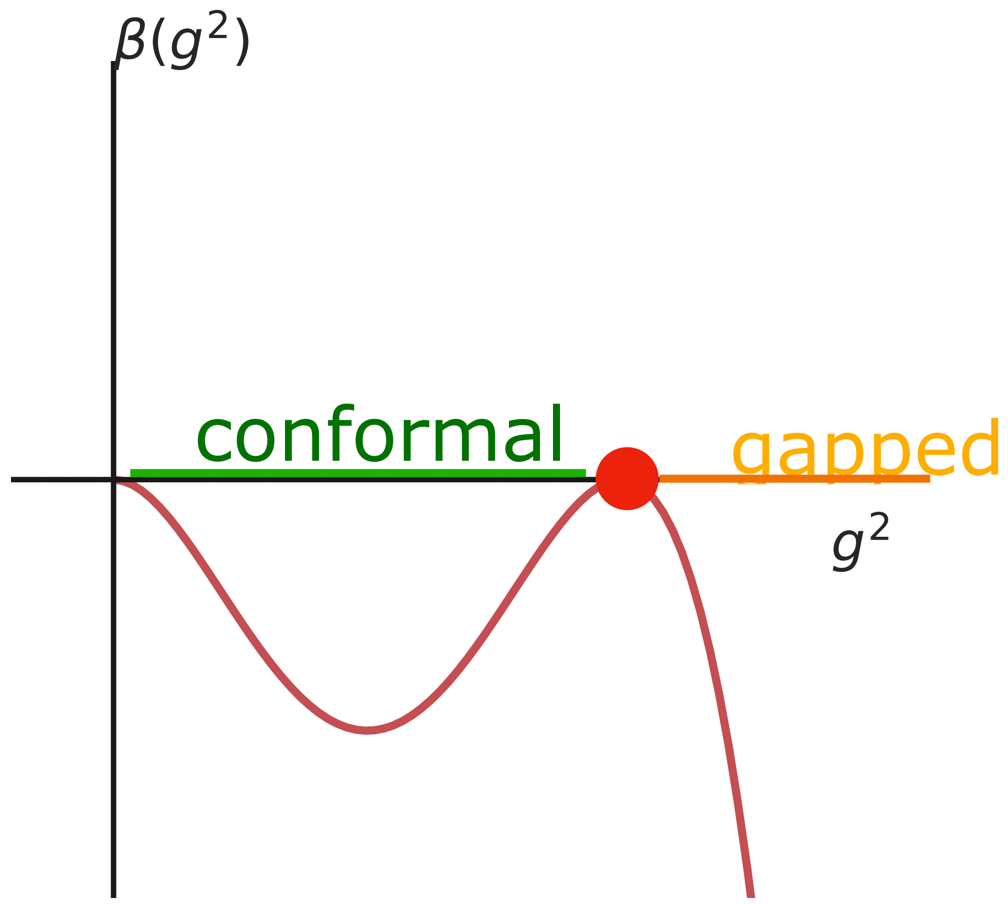
- What can we say about the transition?
  - first order ?
  - ordinary continuous ?
  - BKT-like / merged FP?





Finite size scaling in the critical region

A.H. *Phys.Rev.D* 106 (2022) 1, 014513
for $SU(3), N_f = 8$

FSS describes scaling near the critical point as $\beta \to \beta_*$ in the gapped phase

- The lightest PS mass sets the infinite volume correlation length

  $\xi_\infty(\beta) \propto 1/M_{\mathrm{PS}}^{(\infty)}(\beta)$ (finite in the gapped phase)

- Near criticality L and $\xi_\infty(\beta)$ are the relevant long distance scales

  $$L\,M_{\mathrm{PS}}(\beta, L) = F(L/\xi_\infty(\beta))$$

  is a unique curve, independent of L

- 2nd order scaling: $\xi_\infty \propto |\beta - \beta_*|^{-\nu}$,

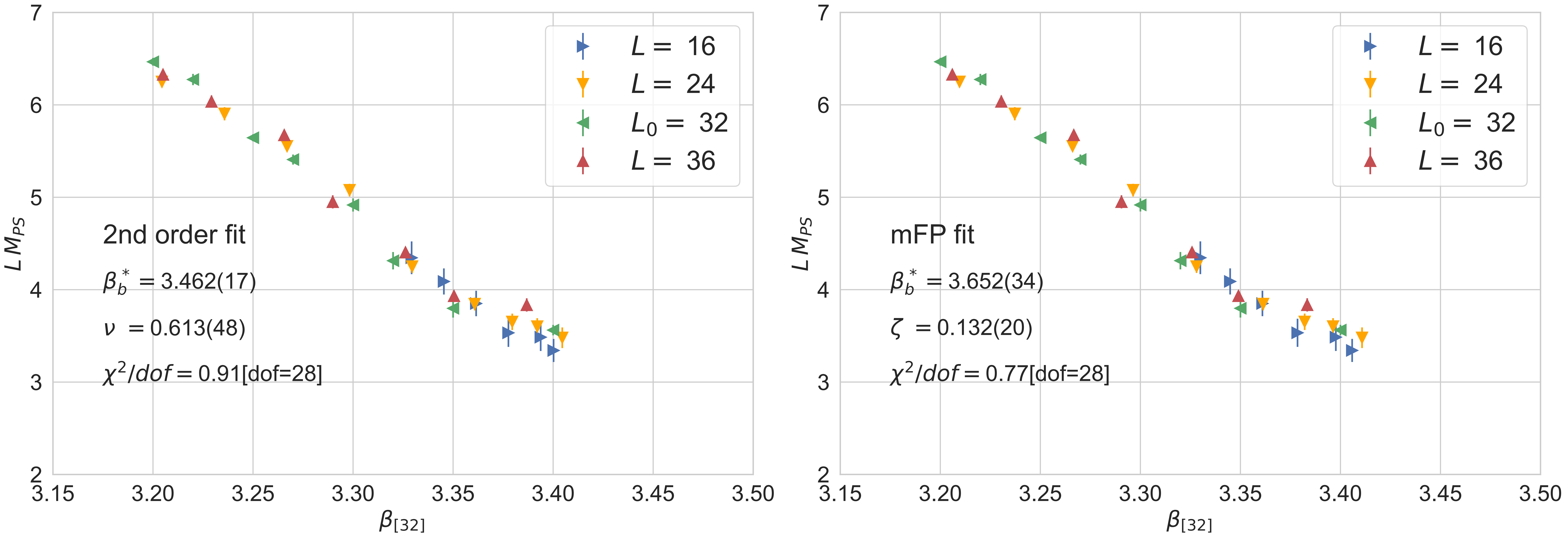
  - 1st order scaling: finite size rounding corresponds to $\nu_{\mathrm{FSS}} = 1/d = 0.25$

- Merged FP (BKT) scaling: $\xi_\infty \propto \exp\left(\dfrac{\zeta}{|\beta/\beta_* - 1|}\right)$

Find the exponents ν or ζ and the infinite volume critical point β_*
by standard curve-collapse analysis

Finite size scaling with $\mathcal{O} = L M_{PS}$

SU(2) + $N_f = 4$



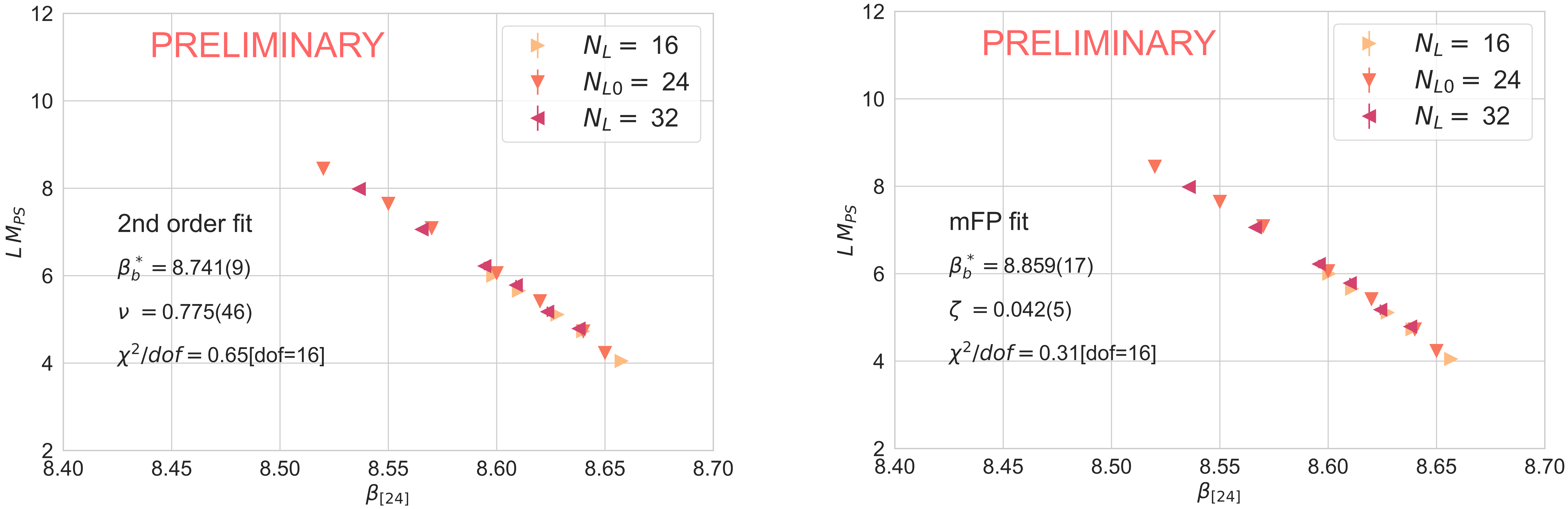
- Both 2nd order and mFP fits show good curve collapse

  $\nu \approx 0.6$ is not consistent with first order transition

- FSS does not (yet) distinguish 2nd order and mFP

Order of the phase transition / FSS :

SU(3) + $N_f = 8$



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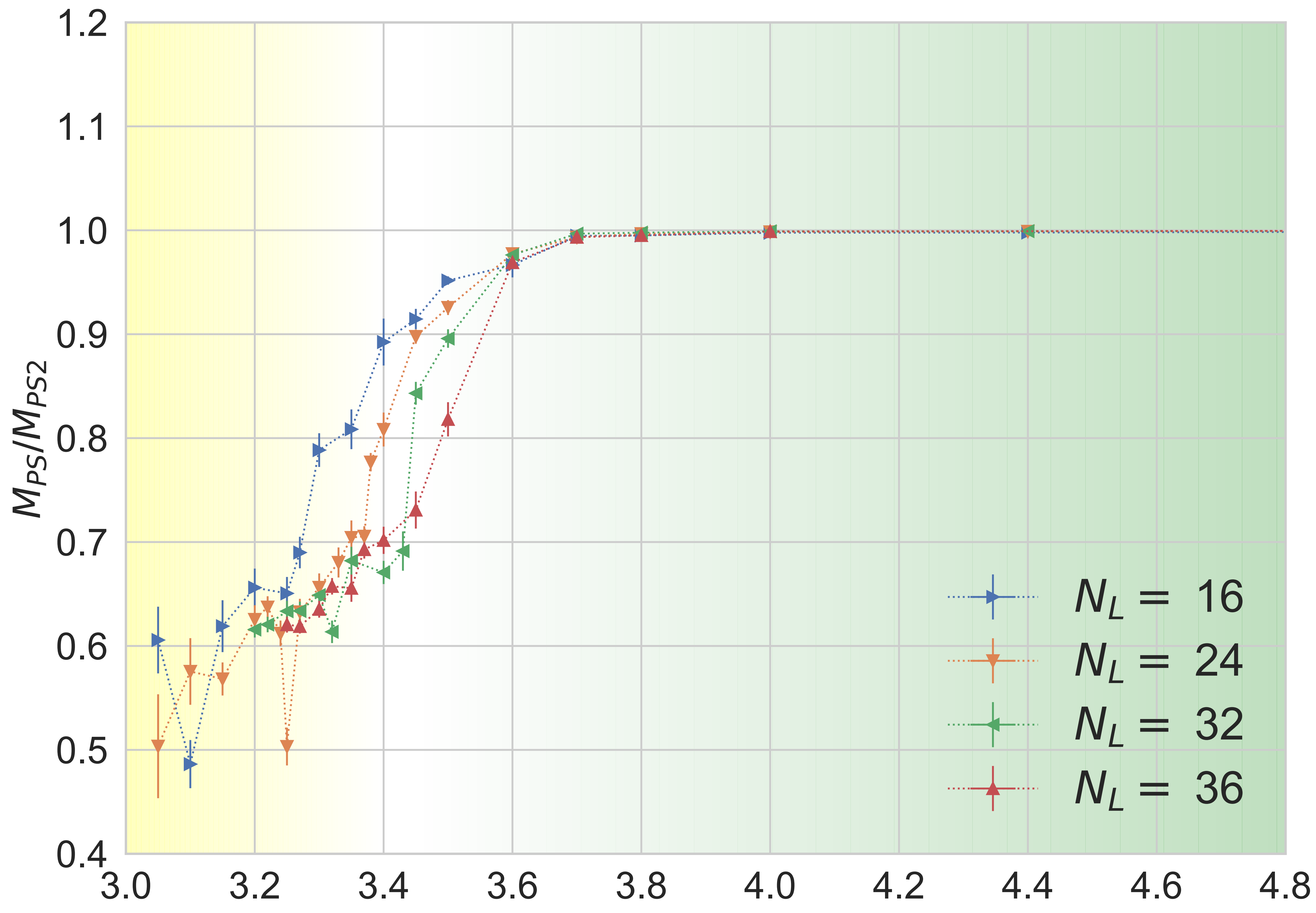
  $\nu \approx 0.76$ is not consistent with first order transition
- FSS does not (yet) distinguish 2nd order and mFP

The taste ratio $R = M_{\rm PS}/M_{\rm PS2}$ - SU(2) + $N_f = 4$

In the infinite volume limit

- conformal phase : $R \to 1$
- gapped phase : $R < 1$
- merged/BKT critical point: $R \to 1$ continuously
- separate 2nd-order UVFP: $R_c \neq 1$ and discontinuity is allowed



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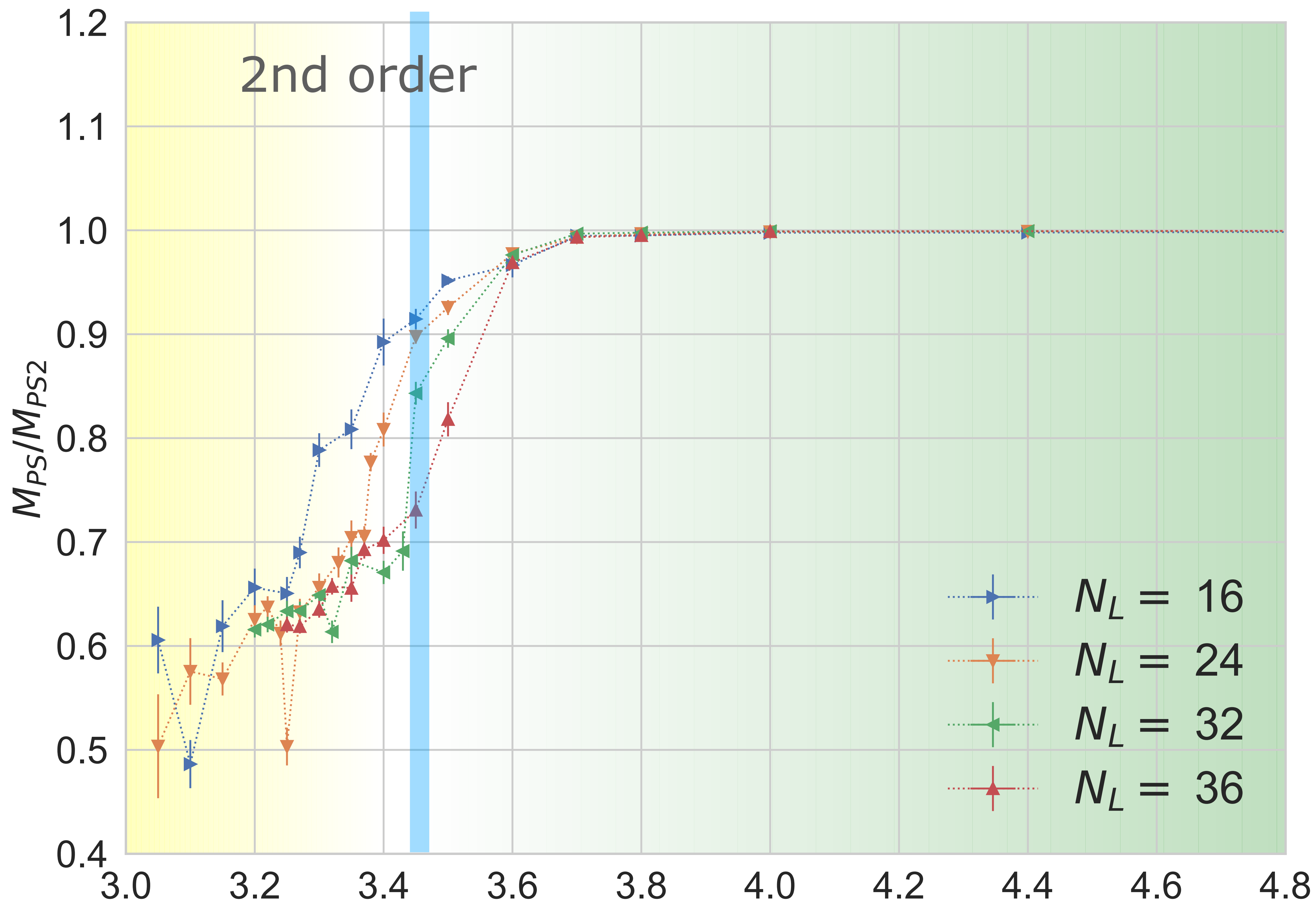

FSS predicts

- 2nd order transition: $\beta_c^{(2nd)} = 3.46(2)$
  - near $\beta = 3.50$ clearly trends below 1
  - **separate 2nd order UVFP is strongly disfavored**

The taste ratio $R = M_{\mathrm{PS}}/M_{\mathrm{PS2}}$ - SU(2) + $N_f = 4$

In the infinite volume limit

- conformal phase : $R \to 1$
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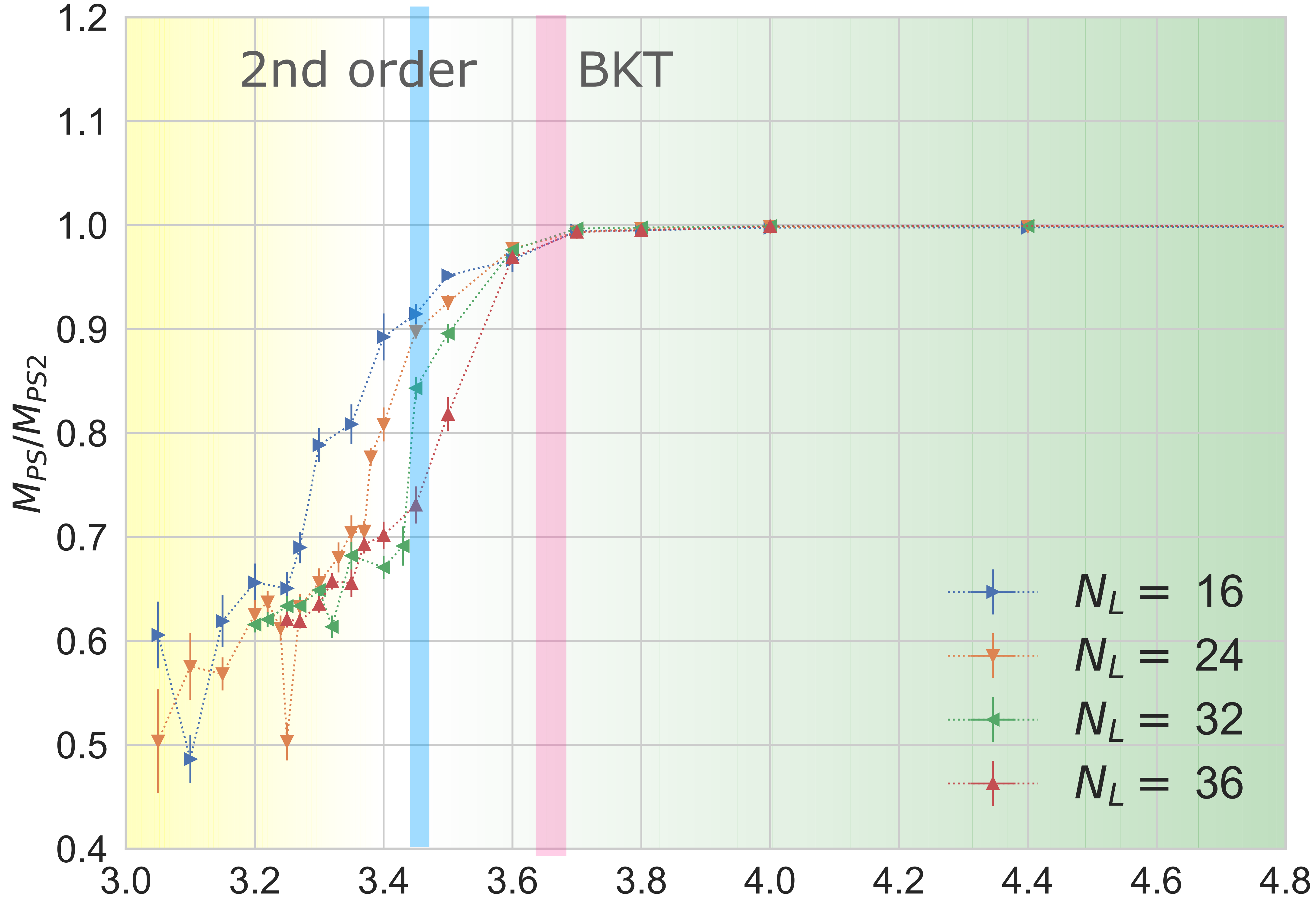

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  - for $\beta \gtrsim \beta_c^{(BKT)}$ $R \simeq 1$
  - **merged/BKT scenario is strongly favored**

The taste ratio $R = M_{\mathrm{PS}}/M_{\mathrm{PS2}}$ - SU(3) + $N_f = 8$

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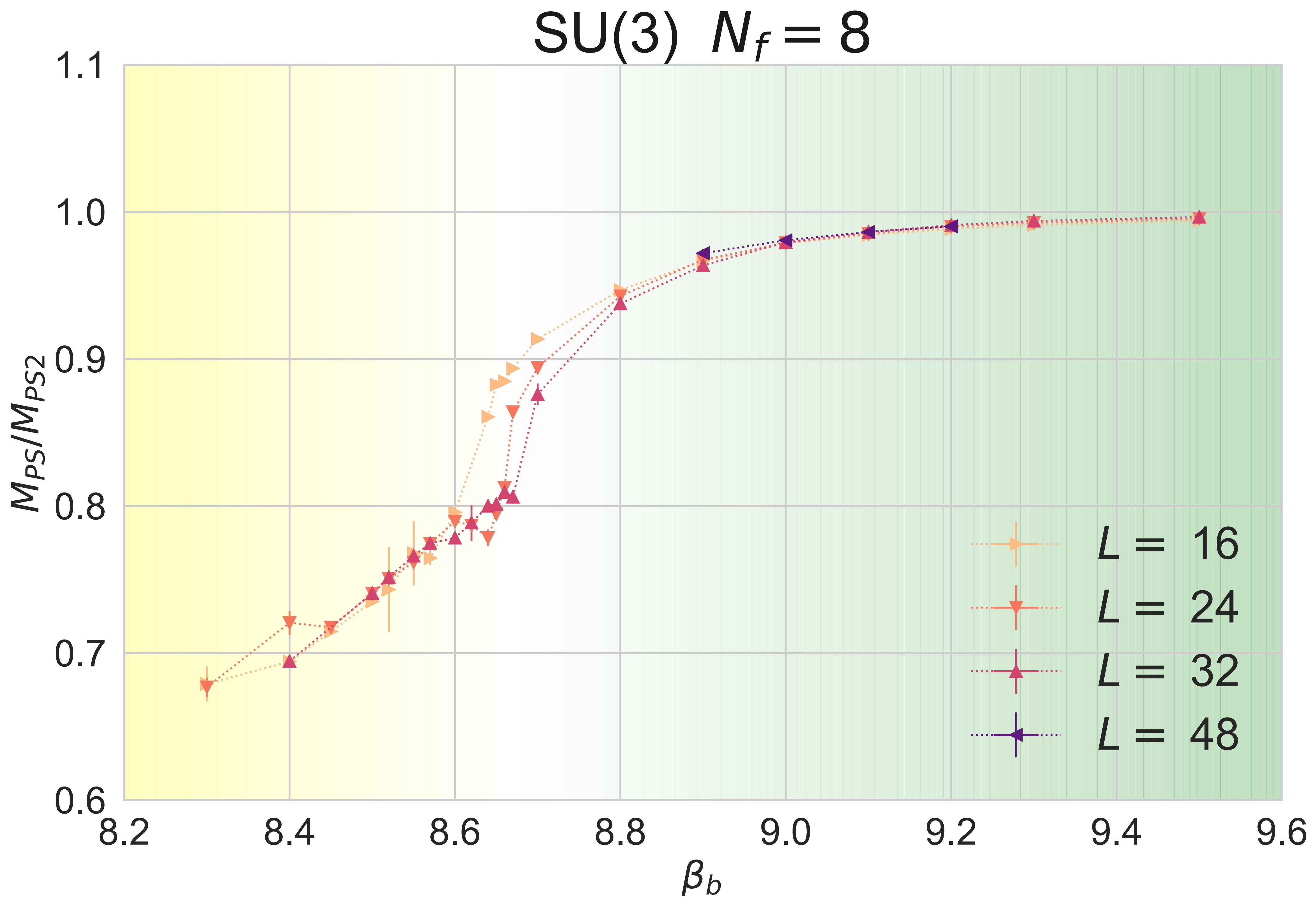

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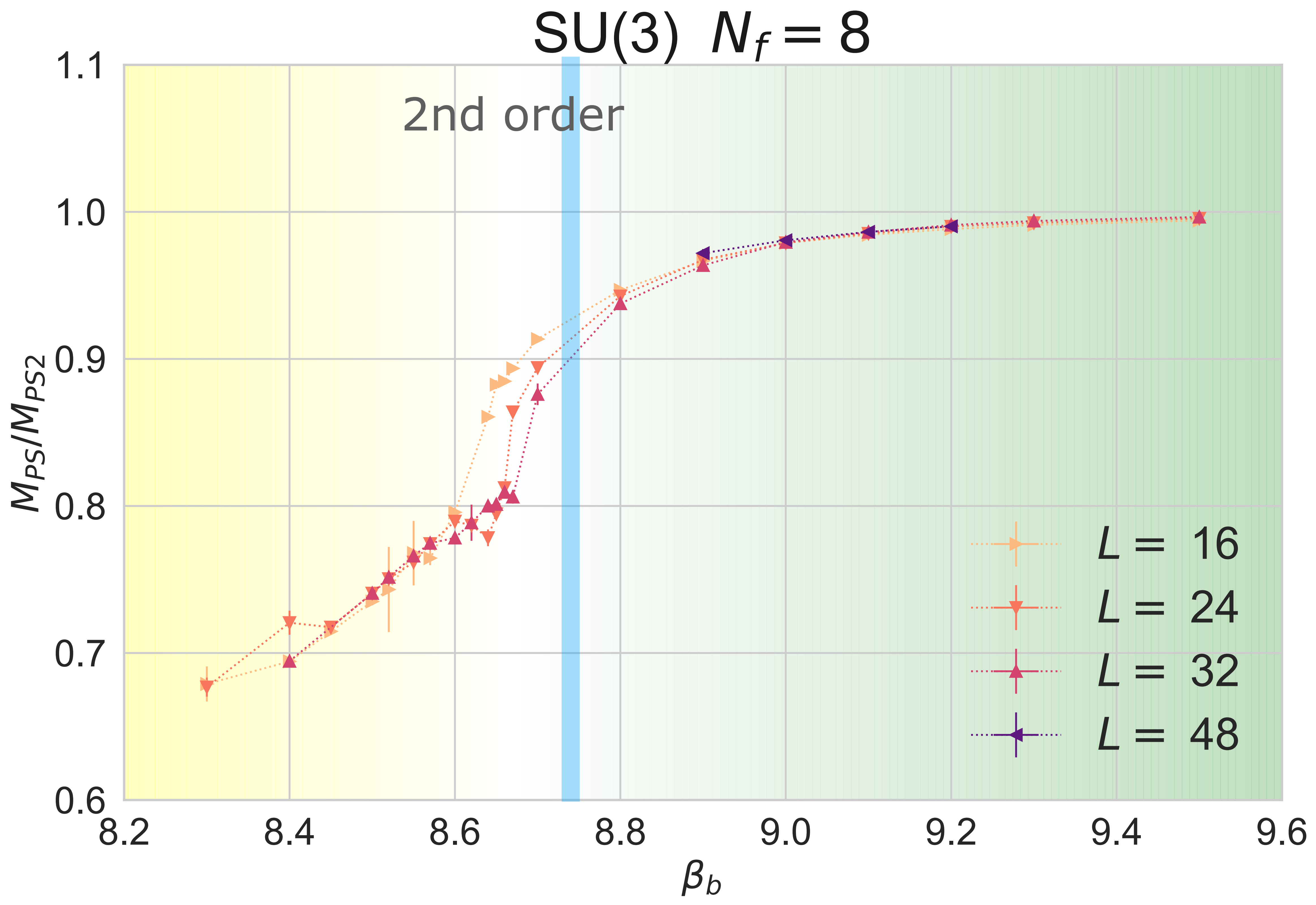

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  - at $\beta \simeq 8.80 > \beta_c^{(2nd)}$ larger L still favors $R < 1$
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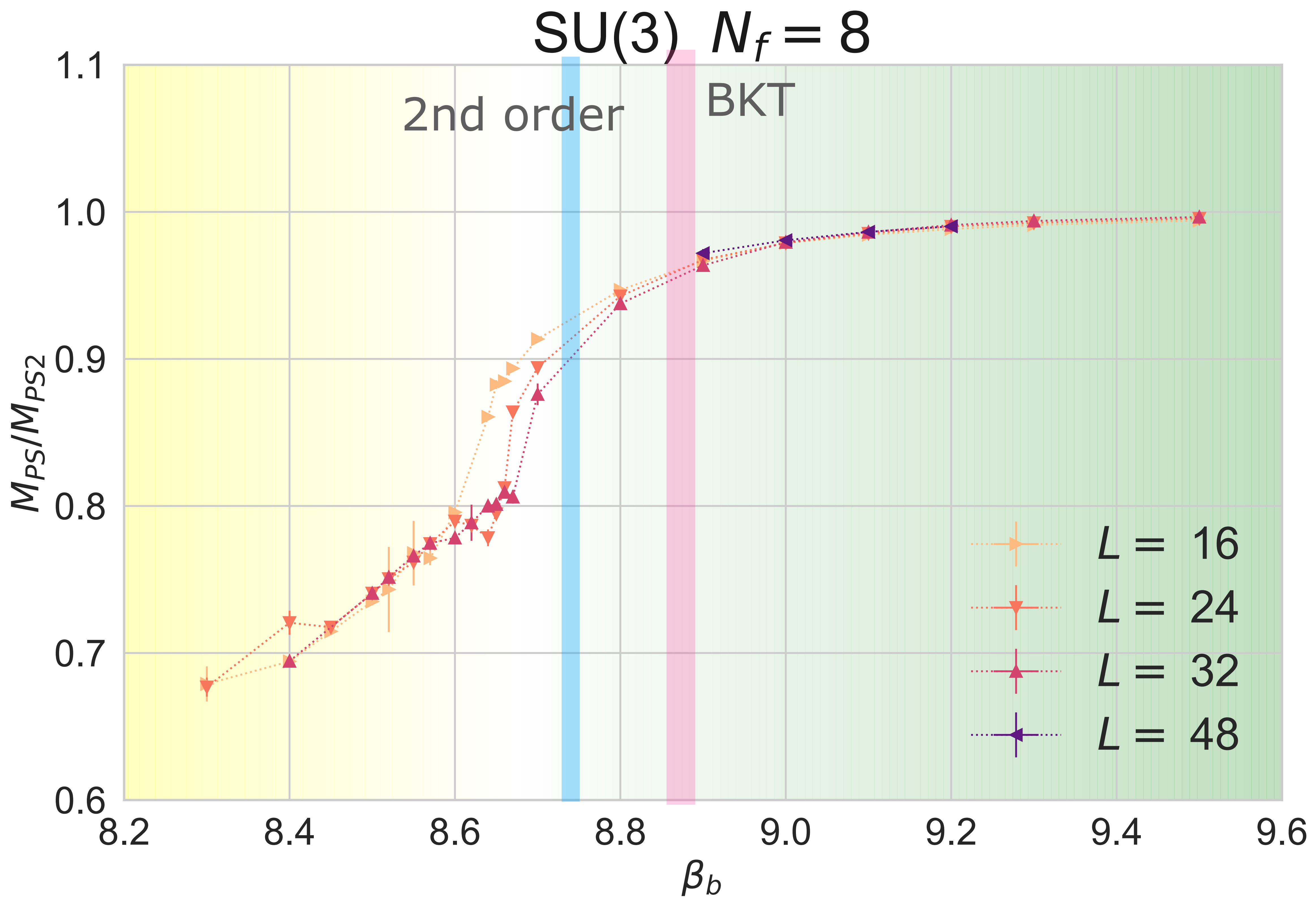

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- BKT-like transition: $\beta_c^{BKT)} = 8.86(2)$
  - for $\beta \gtrsim 9.0$ $R \simeq 1$
  - **merged/BKT scenario is consistent with data**

Finite volume alignment revels very slow RG flow

SU(2) + $N_f = 4$

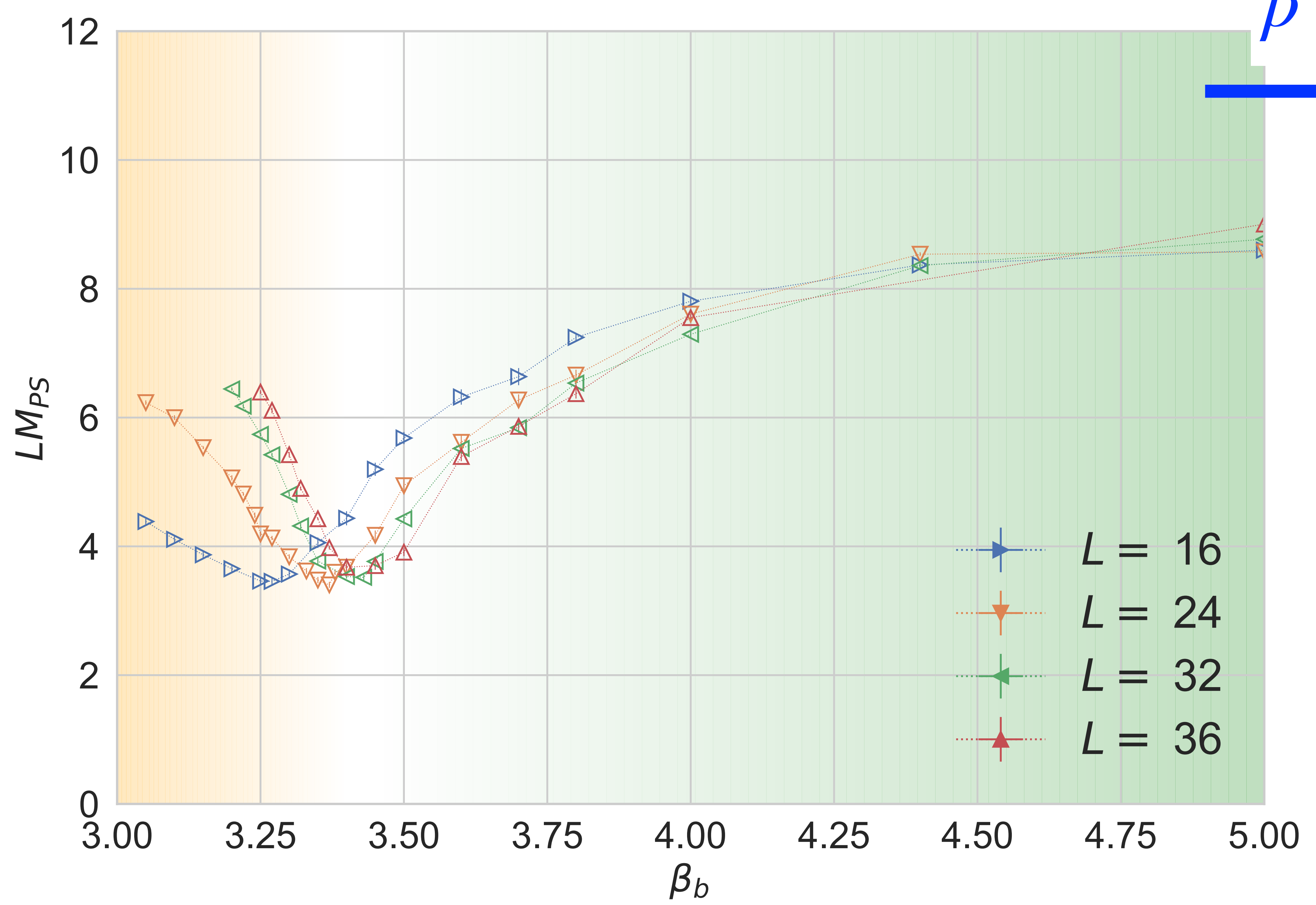
Near the transition $LM_{\rm PS}$ shows strong volume dependence

$\beta_{pc}(L) \neq \beta_*$. (β_{pc} : pseudo-critical coupling)

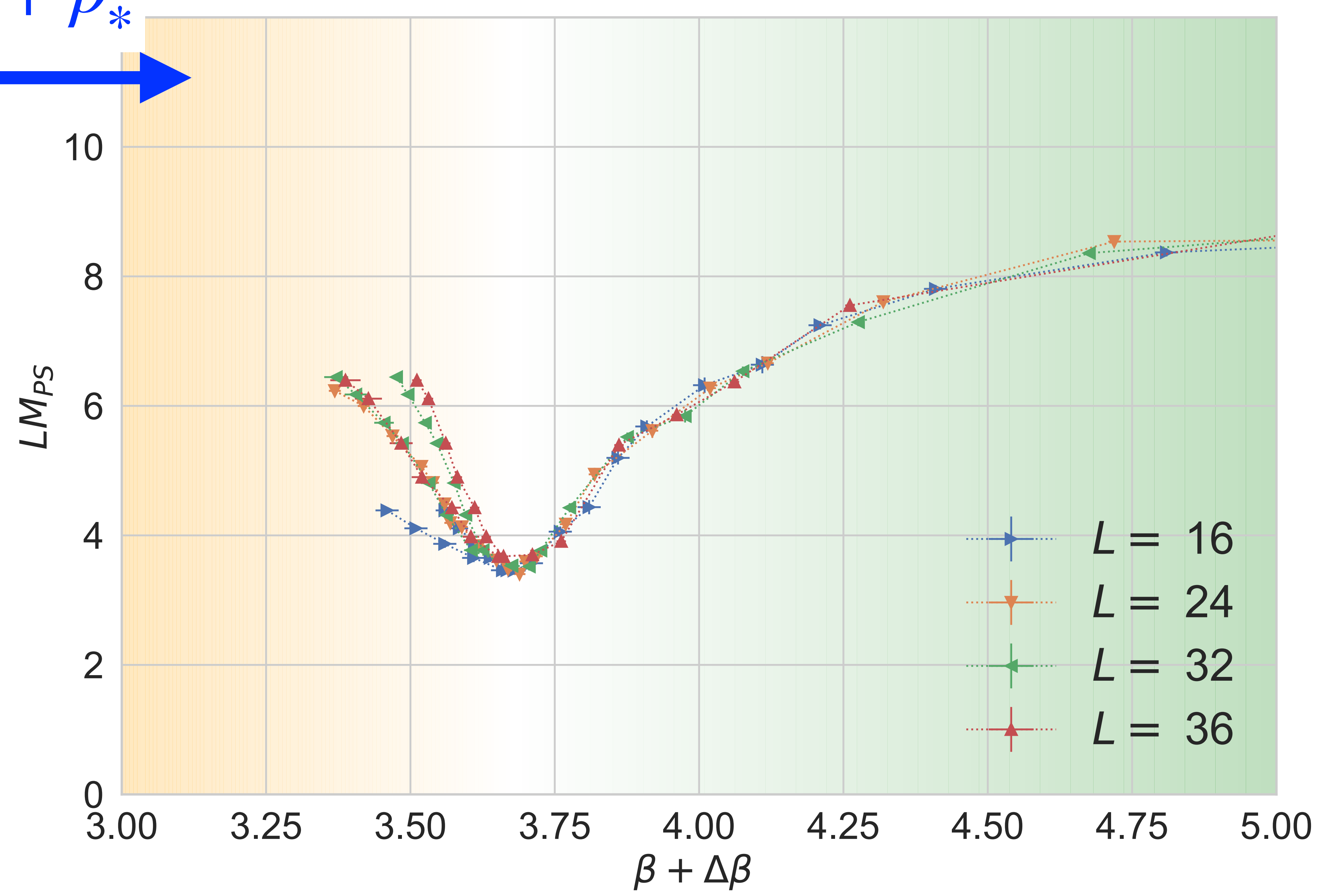
➡ align the critical regions: $\beta(L) \to \beta(L) - \beta_{pc}(L) + \beta_*$

After alignment most volume dependence disappears

➡ **very slow RG running near the critical region**



$\tilde{\beta} = \beta - \beta_{pc}(L) + \beta_*$



Conclusion : A new mass generation paradigm in 4D gauge-fermion systems?

The two “minimal” lattice models

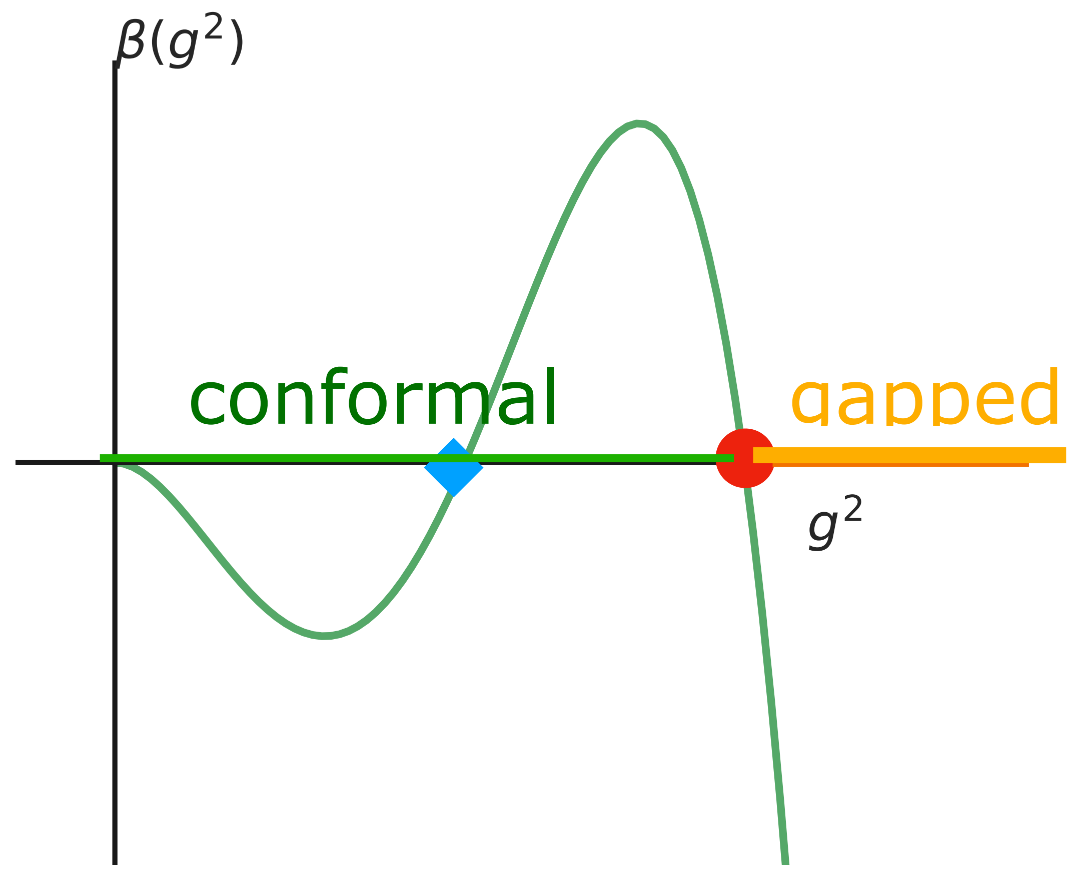
- SU(3) gauge + $N_f = 8$
- SU(2) gauge + $N_f = 4$

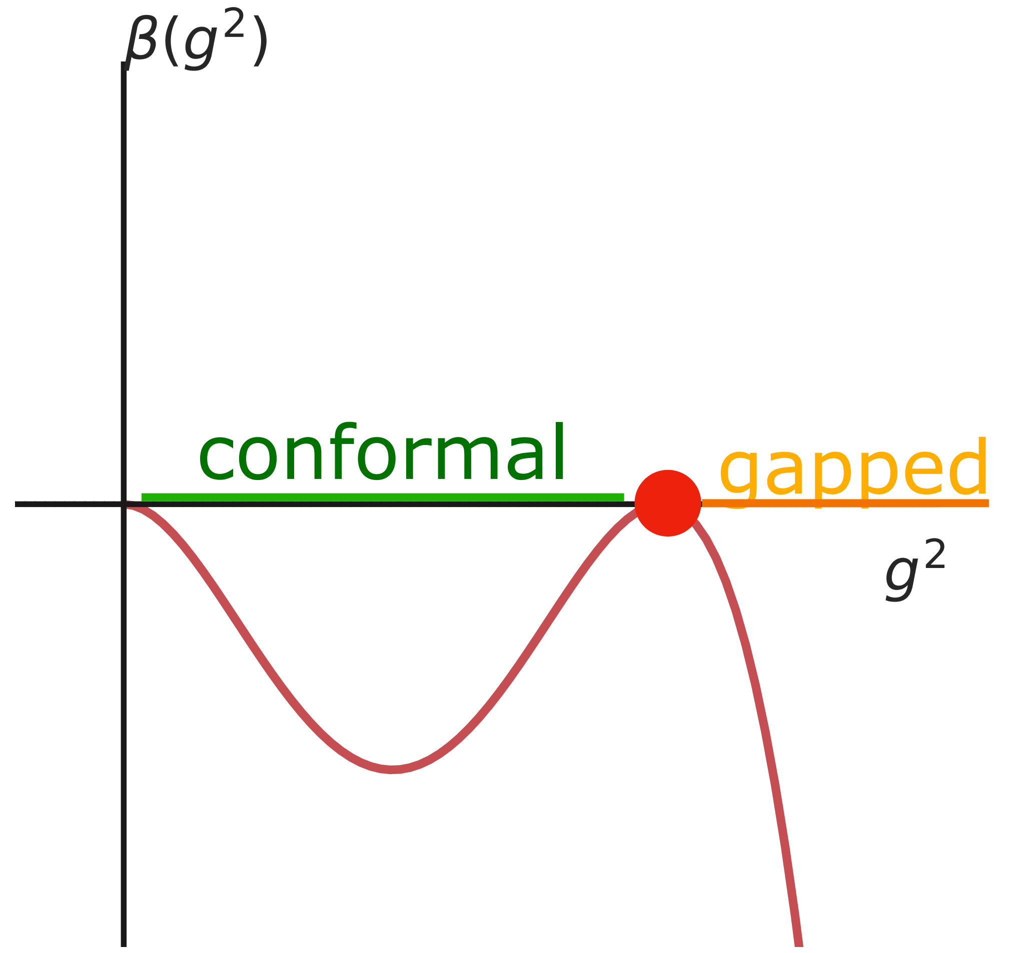
both show

- weak coupling conformal, gapless phase
- strong coupling confining, fully gapped phase
  - no bilinear condensate, no Goldstone bosons
  - shift symmetry (taste symmetry) is broken

Critical behavior:

- 1st order transition is strongly disfavored
- FSS allows both 2nd order or BKT transitions
- ratio $R = M_{\rm PS}/M_{\rm PS2}$
  - SU(2) : separate 2nd order UVFP + IRFP strongly disfavored
  - SU(3) : separate UVFP somewhat disfavored
- Consistent with a very small β function near the critical region





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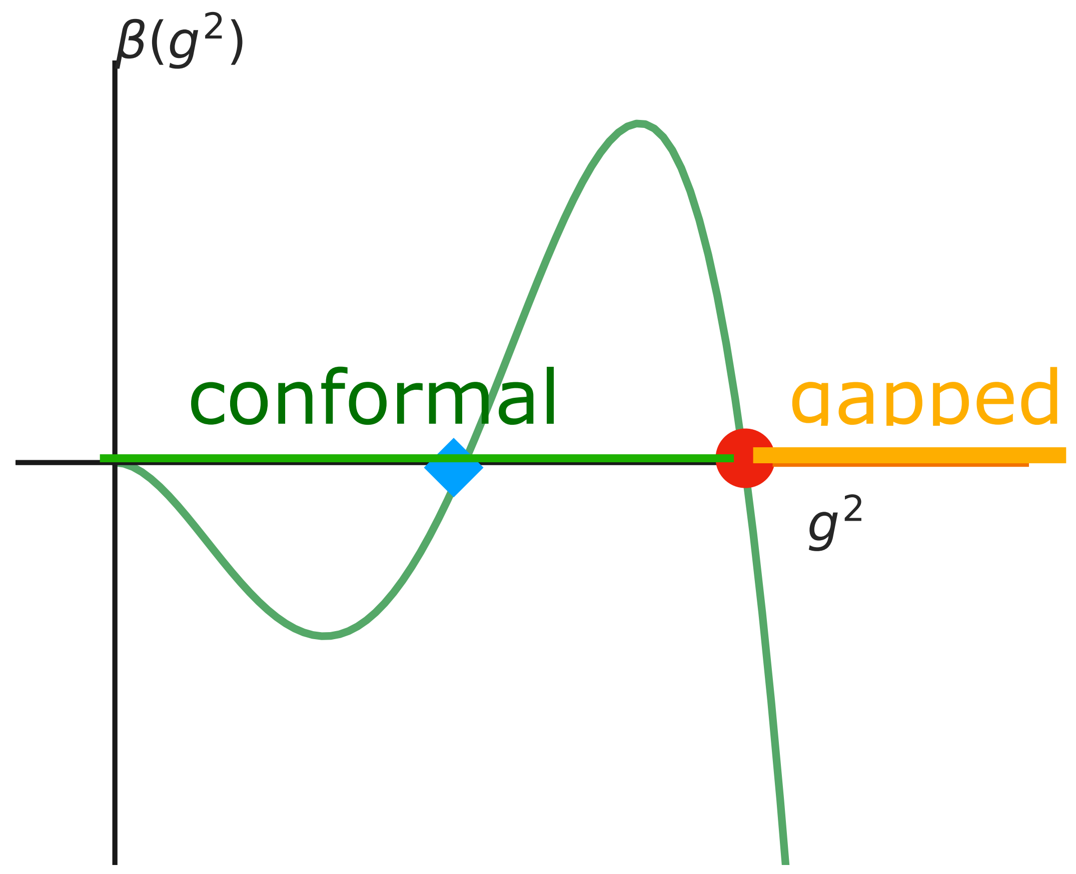
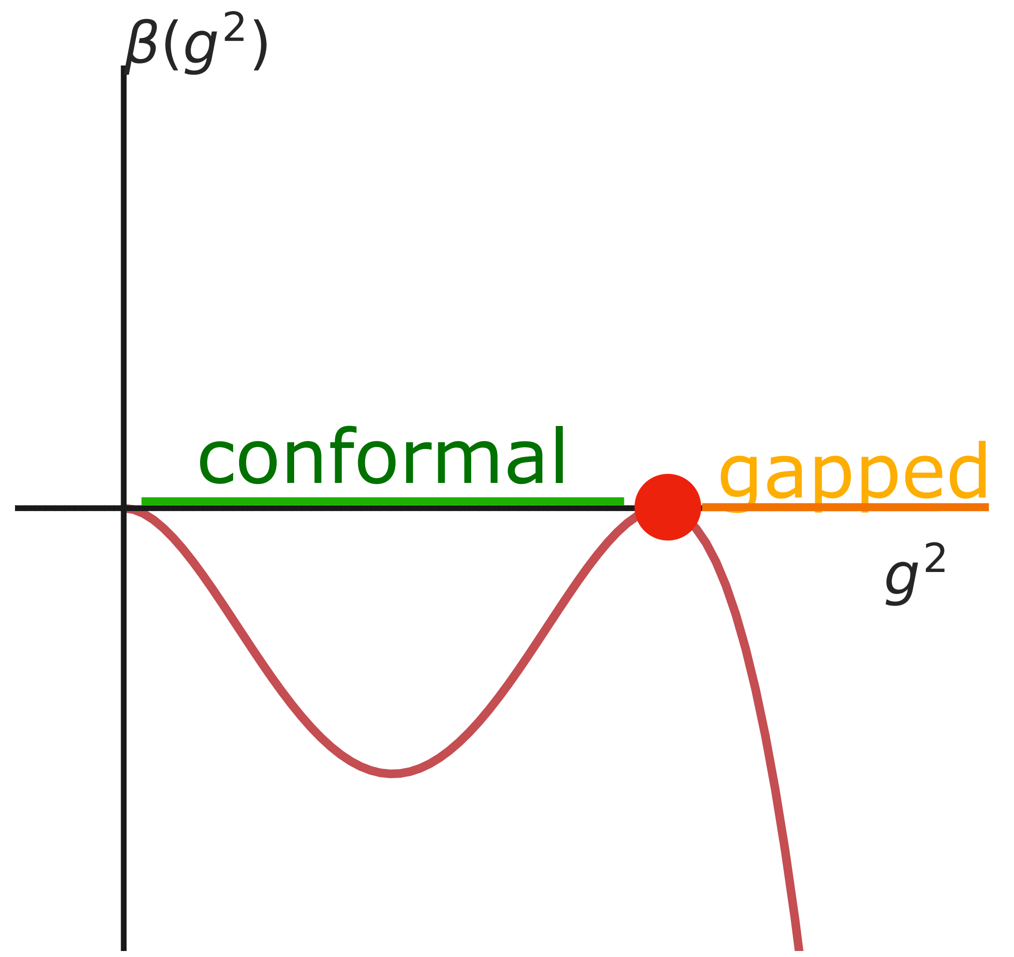
Current evidence favors a merged IR-UV fixed point, but nearby separate fixed points are not excluded.

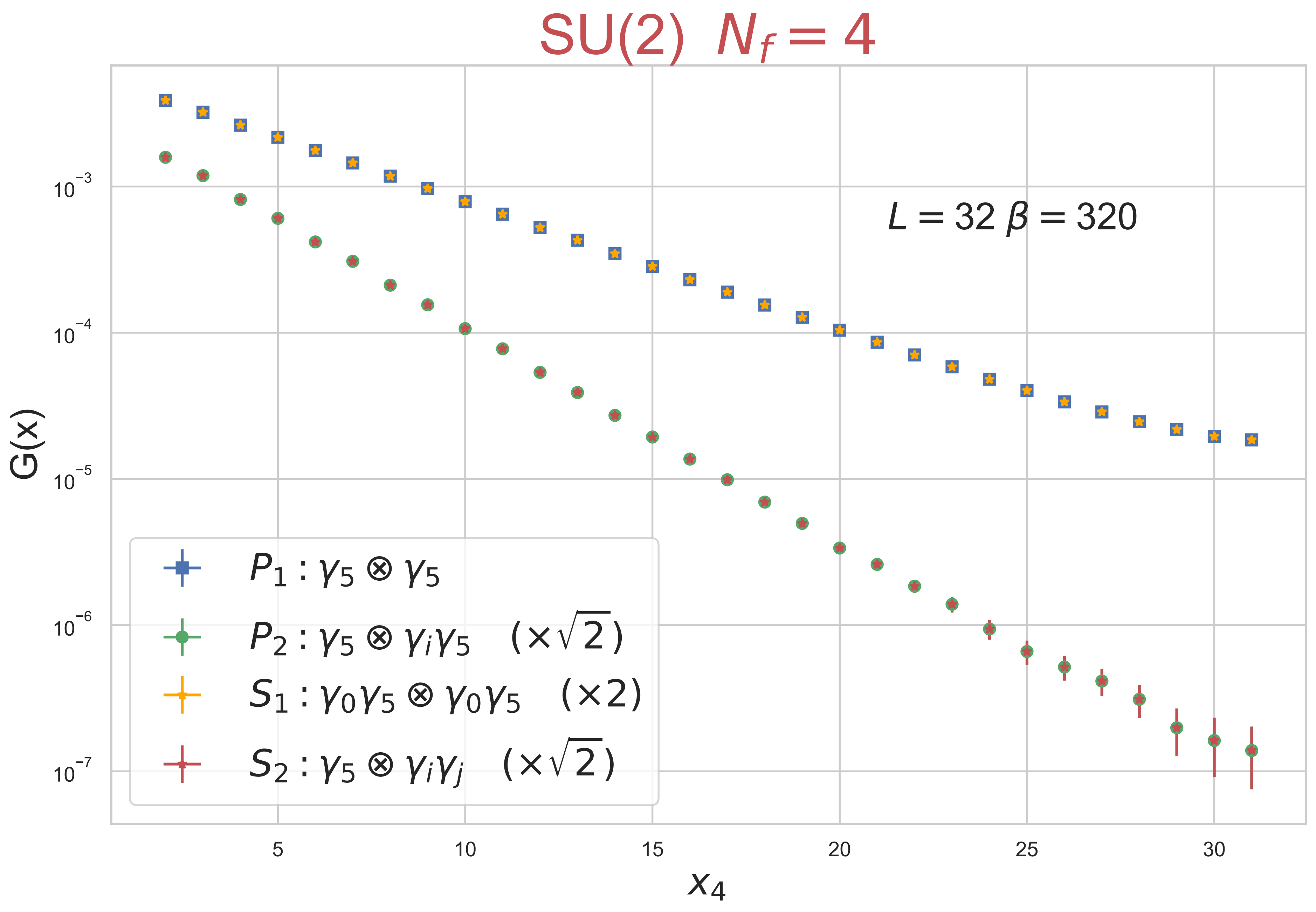




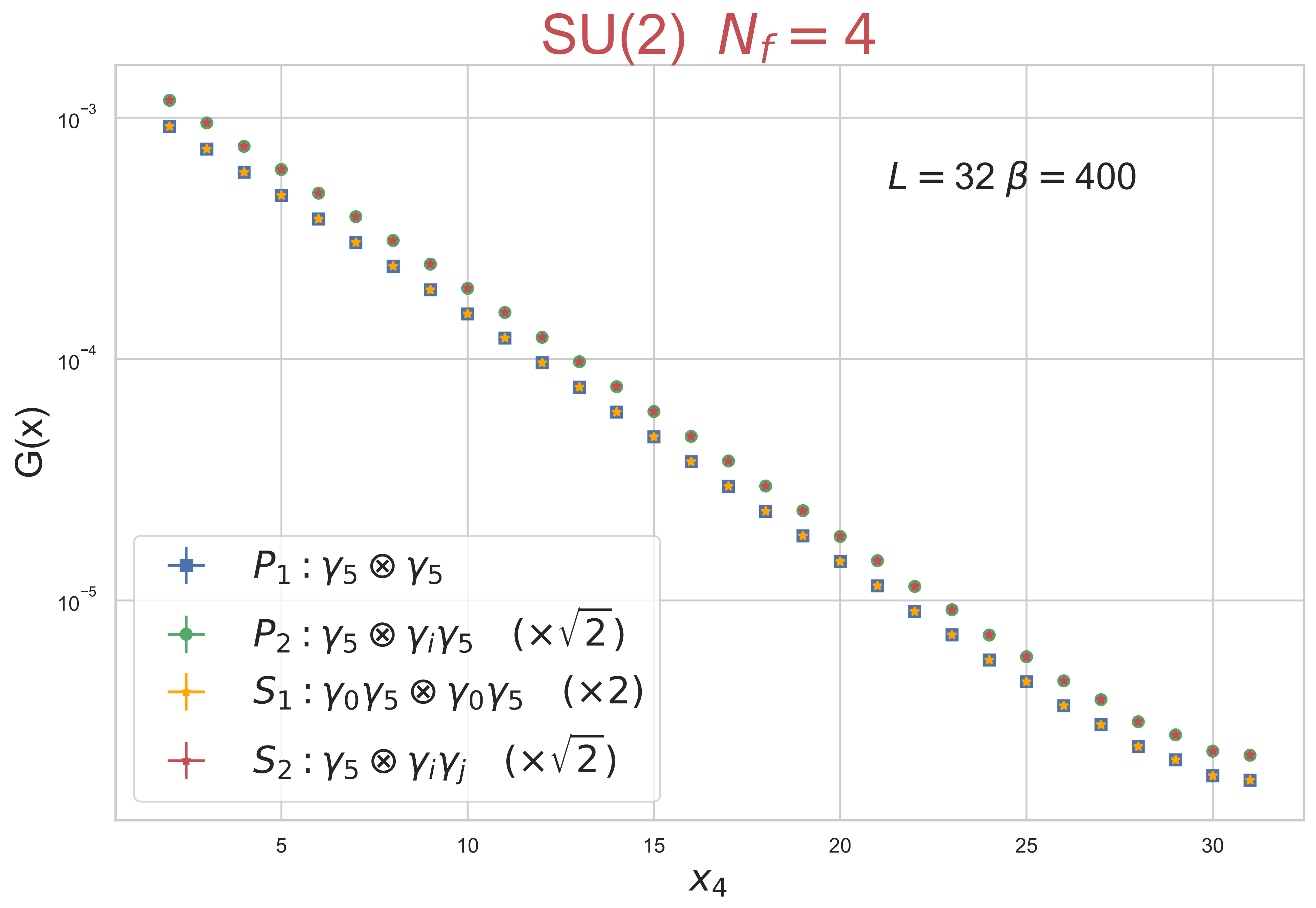
EXTRA SLIDES

No chiral symmetry breaking, but taste breaking

Parity partner correlators ($P_1 - S_1$ and $P_2 - S_2$) are identical in both phases : chiral symmetry to machine precision!



strong coupling phase
significant taste braking



weak coupling phase
approximate taste restoration